Definition of Definite Integral as the Limit of a Sum

Suppose that a function $f(x)$ is continuous on the closed interval $[a,b]$. Divide the interval into n equal subintervals, of length $\Delta x = \frac{b-a}{n}$. Choose one number in each subinterval, in other words, x_1 in the first, x_2 in the second, ..., x_k in the k th,..., and x_n in the n th. Then

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x = \int_a^b f(x)\,dx = F(b) - F(a).$$

Properties of the Definite Integral

Let $f(x)$ and $g(x)$ be continuous on $[a,b]$.

i). $\int_a^b c\cdot f(x)\,dx = c\int_a^b f(x)\,dx$ for any constant c.

ii). $\int_a^a f(x)\,dx = 0$

iii). $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

iv). $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where f is continuous on an interval containing the numbers a, b, and c.

v). If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$

vi). If $f(x)$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

vii). If $f(x) \ge 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge 0$

viii). If $g(x) \ge f(x)$ on $[a,b]$, then $\int_a^b g(x)\,dx \ge \int_a^b f(x)\,dx$