***<u>Dominance and Comparison of Rates of Change</u> **(BC topic only)**
Logarithm functions grow slower than any power function $\left(x^n\right)$.

Among power functions, those with higher powers grow faster than those with lower powers.

All power functions grow slower than any exponential function $\left(a^x,\ a>1\right)$.

Among exponential functions, those with larger bases grow faster than those with smaller bases.

We say, that as $x \to \infty$:

1. $f(x)$ grows <u>faster</u> than $g(x)$ if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$ or if $\lim_{x\to\infty}\frac{g(x)}{f(x)}=0$.

   If $f(x)$ grows faster than $g(x)$ as $x\to\infty$, then $g(x)$ grows <u>slower</u> than $f(x)$ as $x\to\infty$.

2. $f(x)$ and $g(x)$ grow at the <u>same</u> rate as $x\to\infty$ if $\lim_{x\to\infty}\frac{f(x)}{g(x)}=L\neq 0$ (L is finite and nonzero).

For example,

1. e^x grows faster than x^3 as $x\to\infty$ since $\lim_{x\to\infty}\frac{e^x}{x^3}=\infty$
2. x^4 grows faster than $\ln x$ as $x\to\infty$ since $\lim_{x\to\infty}\frac{x^4}{\ln x}=\infty$
3. x^2+2x grows at the same rate as x^2 as $x\to\infty$ since $\lim_{x\to\infty}\frac{x^2+2x}{x^2}=1$

To find some of these limits as $x\to\infty$, you may use the graphing calculator. Make sure that an appropriate viewing window is used.