32. ***Logistics (BC topic)

1. Rate is jointly proportional to its size and the difference between a fixed positive number (L) and its size.

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right) \text{ OR } \frac{dy}{dt} = ky(M - y) \text{ which yields}$$

$$y = \frac{L}{1 + Ce^{-kt}} \quad \text{through separation of variables}$$

2. $\lim_{t\to\infty} y = L$; L = carrying capacity (Maximum); horizontal asymptote

3. y-coordinate of inflection point is $\frac{L}{2}$, i.e. when it is growing the fastest (or max rate).

32(a). ***Decomposition:

Steps:

1. Use Long Division first if the degree of the Numerator is equal or more than the Denominator to get $\int \frac{N(x)}{D(x)}dx = \int q(x)dx + \int \frac{r(x)}{D(x)}dx$
2. For the second integral, factor $D(x)$ completely into Linear factors to get $\frac{r(x)}{D(x)} = \frac{A}{linearfactor\#1} + \frac{B}{linearfactor\#2} + \ldots$
3. Multiply both sides by $D(x)$ to eliminate the fractions
4. Choose your x-values wisely so that you can easily solve for A, B, C, etc
5. Rewrite your integral that has been decomposed and integrate everything.

Logistic

$$\frac{dP}{dt} = \frac{k}{M}P(M - P)$$

$$P = \frac{M}{1 + Ce^{-kt}}$$

M = carrying capacity