***<u>Parametric Form of the Derivative</u>

If a smooth curve **C** is given by the parametric equations $x = f(x)$ and $y = g(t)$, then the slope of the curve **C** at (x, y) is $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, $\frac{dx}{dt} \neq 0$.

Note: The second derivative, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{dy}{dx}\right] \div \frac{dx}{dt}$.

***<u>Arc Length in Parametric Form</u>

If a smooth curve **C** is given by $x = f(t)$ and $y = g(t)$ and these functions have continuous first derivatives with respect to t for $a \leq t \leq b$, and if the point $P(x, y)$ traces the curve exactly once as t moves from $t = a$ to $t = b$, then the length of the curve is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{\left(f'(t)\right)^2 + \left(g'(t)\right)^2}\, dt.$$

$$\text{speed} = \sqrt{\left(f'(t)\right)^2 + \left(g'(t)\right)^2}$$

Arc Length $= \underbrace{\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}}_{\text{cartesian}}\, dx$ **Arc Length** $= \underbrace{\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt}_{\text{parametric}}$

Speed = $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ **T.D.T.** = $\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Polar Area $= \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$ **Parametric Derivatives:** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$, $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Polar Conversions: $r^2 = x^2 + y^2$, $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \arctan\frac{y}{x}$