***Polar Coordinates

1. Cartesian vs. Polar Coordinates. The polar coordinates (r, θ) are related to the Cartesian coordinates (x, y) as follows:

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \text{ and } x^2 + y^2 = r^2$$

2. To find the points of intersection of two polar curves, find (r, θ) satisfying the first equation for which some points $(r, \theta + 2n\pi)$ or $(-r, \theta + \pi + 2n\pi)$ satisfy the second equation. Check separately to see if the origin lies on both curves, i.e. if r can be 0. Sketch the curves.

3. Area in Polar Coordinates: If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by

$$A = \frac{1}{2}\int_{\alpha}^{\beta} \left(f(\theta)\right)^2 d\theta = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$$

4. Derivative of Polar function: Given $r = f(\theta)$, to find the derivative, use parametric equations.

$$x = r\cos\theta = f(\theta)\cos\theta \quad \text{and} \quad y = r\sin\theta = f(\theta)\sin\theta.$$

Then $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

5. Arc Length in Polar Form: $$s = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$