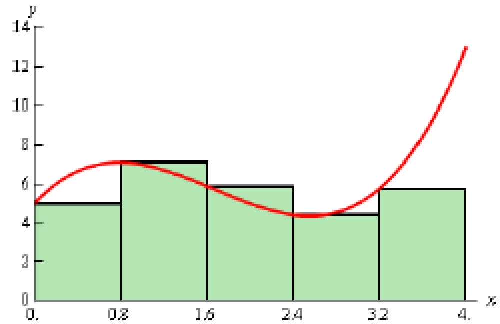
Numerical Methods

Riemann Sum



$$P_0(x) = \int_a^b f(x)\,dx = \lim_{\|P\|\to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

where $a = x_0 < x_1 < x_2 < \cdots < x_n = b$

and $\Delta x_i = x_i - x_{i-1}$

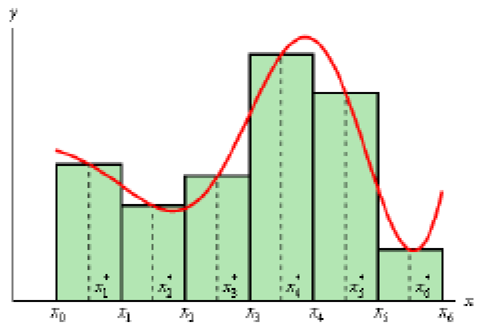
and $\|P\| = max\{\Delta x_i\}$

Types:

- Left Sum (LHS)
- Middle Sum (MHS)
- Right Sum (RHS)

Numerical Methods

Midpoint Rule
(Middle Sum)



$$P_0(x) = \int_a^b f(x)\,dx \;\approx \sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x =$$

$$\Delta x[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \cdots + f(\bar{x}_n)]$$

where $\Delta x = \frac{b-a}{n}$

and $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = midpoint\ of\ [x_{i-1}, x_i]$

Error Bounds: $|E_M| \leq \frac{K(b-a)^3}{24n^2}$

Riemann Sums:

$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$ $\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\Delta x$ $\Delta x = \frac{b-a}{n}$
$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sum_{i}(\text{height of } i\text{th rectangle})\cdot(\text{width of } i\text{th rectangle})$ Right Endpoint Rule: $\sum_{i=1}^{n} f(a+i\Delta x)(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a+i\frac{(b-a)}{n})$ Left Endpoint Rule: $\sum_{i=1}^{n} f(a+(i-1)\Delta x)(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a+(i-1)\frac{(b-a)}{n})$ Midpoint Rule: $\sum_{i=1}^{n} f(a+\left(\frac{(i-1)+i}{2}\right)\Delta x)(\Delta x) = \sum_{i=1}^{n} (\frac{(b-a)}{n}) f(a+\left(\frac{(i-1)+i}{2}\right)\frac{(b-a)}{n})$