

NOTE: These tests prove convergence and divergence, not the actual limit L or sum S .

Sequence: $\lim_{n \rightarrow \infty} a_n = L$
($a_n, a_{n+1}, a_{n+2}, \dots$)

Series: $\sum_{n=1}^{\infty} a_n = S$
($a_n + a_{n+1} + a_{n+2} + \dots$)

***Sequences and Series

1. If a sequence $\{a_n\}$ has a limit L , that is, $\lim_{n \rightarrow \infty} a_n = L$, then the sequence is said to converge to L . If there is no limit, the series diverges. If the sequence $\{a_n\}$ converges, then its limit is unique. Keep in mind that

$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$; $\lim_{n \rightarrow \infty} x^{\left(\frac{1}{n}\right)} = 1$; $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$; $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$. These limits are useful and arise frequently.

2. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges; the geometric series $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $|r| < 1$ and diverges if $|r| \geq 1$ and $a \neq 0$.

3. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Sequences & Series	
Sequence	$\lim_{n \rightarrow \infty} a_n = L \text{ (Limit)}$ <p>Example: $(a_n, a_{n+1}, a_{n+2}, \dots)$</p>
Geometric Series	$S = \lim_{n \rightarrow \infty} \frac{a(1 - r^n)}{1 - r} = \frac{a}{1 - r}$ <p style="text-align: center;">only if $r < 1$ where r is the radius of convergence and $(-r, r)$ is the interval of convergence</p>