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Alternating Series Test

Series: $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

<u>Condition of Convergence:</u>

$$0 < a_{n+1} \le a_n$$
$$\lim_{n\to\infty} a_n = 0$$

or if $\sum_{n=0}^{\infty} |a_n|$ is convergent

<u>Condition of Divergence:</u>

None. This test cannot be used to show divergence.

* Remainder: $|R_n| \le a_{n+1}$

Tests for Convergence/Divergence

n^{th} term test	div. if $\lim_{n\to\infty} a_n \neq 0$ (cannot be used to show convergence)
Geom. series test	$\sum_{n=0}^{\infty} ar^n$ $\quad \lvert r\rvert < 1 \to$ conv. , $\lvert r\rvert \geq 1 \to$ div. , $S = \frac{a}{1-r}$
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\quad p > 1 \to$ conv. , $p \leq 1 \to$ div.
Alternating series	decr. terms and $\lim_{n\to\infty} a_n = 0 \to$ conv.
Integral test	$a_n = f(x)$ $\quad \sum_{n=1}^{\infty} a_n$ conv. if $\int_1^{\infty} f(x)\,dx$ conv., $\sum_{n=1}^{\infty} a_n$ div. if $\int_1^{\infty} f(x)\,dx$ div.
Ratio test	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1 \to$ conv. , $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1 \to$ div. , (inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$) (works well for factorials and exponentials)

Convergence Tests		
Series Convergence Tests	1. Divergence or n^{th} Term 2. Geometric Series 3. p-Series 4. Alternating Series 5. Integral	6. Ratio 7. Root 8. Direct Comparison 9. Limit Comparison 10. Telescoping Series