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Limit Comparison Test

$(\{a_n\}, \{b_n\} > 0)$

Series: $\sum_{n=1}^{\infty} a_n$

Condition of Convergence:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$

and $\sum_{n=0}^{\infty} b_n$ converges

Condition of Divergence:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$$

and $\sum_{n=0}^{\infty} b_n$ diverges

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ c finite & $a_n, b_n > 0$? — YES

Does $\sum_{n=1}^{\infty} b_n$ converge?

YES → $\sum a_n$ Converges

NO → $\sum a_n$ Diverges

Limit Comparison

if $\lim_{n\to\infty} \frac{a_n}{b_n}$ is finite and positive both series converge or both diverge

(use with "messy" algebraic series, usually compared to a p-series)

Limit Comparison Test: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be a series of nonnegative terms, with $a_n \neq 0$ for all sufficiently large n, and suppose that $\lim_{n\to\infty} \frac{b_n}{a_n} = c > 0$. Then the two series either both converge or both diverge.