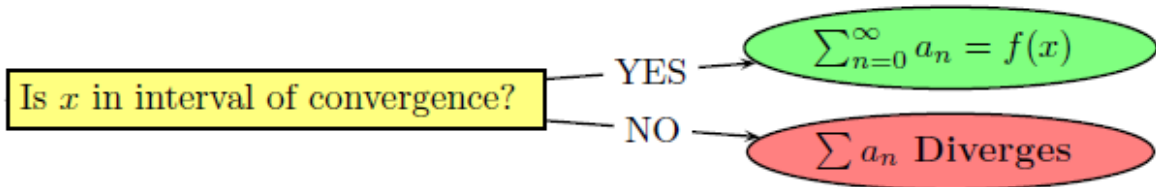


TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!} (x - a)^n$? — YES



Taylor Series: Let f be a function with derivatives of all orders throughout some interval containing a as an interior point. Then the Taylor series generated by f at a is

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x - a)^k = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!} (x - a)^2 + \dots + \frac{f^{(n)}(a)}{n!} (x - a)^n + \dots$$

The remaining terms after the term containing the n th derivative can be expressed as a remainder to Taylor's Theorem:

$$f(x) = f(a) + \sum_{k=1}^n \frac{f^{(k)}(a)}{k!} (x - a)^k + R_n(x) \text{ where } R_n(x) = \frac{1}{n!} \int_a^x (x - t)^n f^{(n+1)}(t) dt$$

Lagrange's form of the remainder: $|f(x) - P_n(x)| = |R_n(x)| = \frac{f^{(n+1)}(c) |x - a|^{n+1}}{(n+1)!}$

, where $a < c < x$.

The series will converge for all values of x for which the remainder approaches zero as $x \rightarrow \infty$.