Volume of Solids with Known Cross Sections

1. For cross sections of area $A(x)$, taken perpendicular to the x-axis, volume = $\int_a^b A(x)\,dx$.

Cross-sections {if only one function is used then just use that function, if it is between two functions use *top-bottom if perpendicular to the x-axis or right-left if perpendicular to the y-axis*} mostly all the same only varying by a constant, with the only exception being the rectangular cross-sections:

- Square cross-sections:
  $$V=\int_a^b (top\ function - bottom\ function)^2\,dx$$
- Equilateral cross-sections:
  $$V=\frac{\sqrt{3}}{4}\int_a^b (top\ function - bottom\ function)^2\,dx$$
- Isosceles Right Triangle cross-sections (hypotenuse in the xy plane):
  $$V=\frac{1}{4}\int_a^b (top\ function - bottom\ function)^2\,dx$$

- Isosceles Right Triangle cross-sections (leg in the *xy* plane):

$$V = \frac{1}{2}\int_a^b \left(top\ function - bottom\ function\right)^2\, dx$$

- Semi-circular cross-sections:

$$V = \frac{\pi}{8}\int_a^b \left(top\ function - bottom\ function\right)^2\, dx$$

- Rectangular cross-sections (height function or value must be given or articulated somehow – notice no "square" on the {top – bottom} part):

$$V = \int_a^b \left(top\ function - bottom\ function\right)\bullet(height\ function\,/\,value)dx$$

- Circular cross-sections with the diameter in the *xy* plane:

$$V = \frac{\pi}{4}\int_a^b \left(top\ function - bottom\ function\right)^2\, dx$$

- Square cross-sections with the diagonal in the *xy* plane:

$$V = \frac{1}{2}\int_a^b \left(top\ function - bottom\ function\right)^2\, dx$$

For cross sections of area $A(y)$, taken perpendicular to the y-axis, volume = $\int_a^b A(y)\, dy$.

Volume of Solids of Revolution (rectangles drawn perpendicular to the axis of revolution)

- Revolving around a horizontal line (y=# or x-axis) where $a \le x \le b$:

  Axis of Revolution and the region being revolved:

$$V = \pi\int_a^b (furthest\ from\ a.r. - a.r.)^2 - (closest\ to\ a.r. - a.r.)^2\,dx$$

- Revolving around a vertical line (x=# or y-axis) where $c \le y \le d$ (or use Shell Method):

  Axis of Revolution and the region being revolved:

$$V = \pi\int_c^d (furthest\ from\ a.r. - a.r.)^2 - (closest\ to\ a.r. - a.r.)^2\,dy$$

***Shell Method (used if function is in terms of x and revolving around a vertical line) where $a \le x \le b$:

$$V = 2\pi \int_a^b r(x)h(x)dx$$

$r(x) = x$ if a.r. is y-axis ($x = 0$)

$r(x) = (x - a.r.)$ if a.r. is to the left of the region

$r(x) = (a.r. - x)$ if a.r. is to the right of the region

$h(x) = f(x)$ if only revolving with one function

$h(x) = (top - bottom)$ if revolving the region between two functions

Volume **Disc** $V = \pi \int_a^b r^2 \, dx$	**Washer** $V = \pi \int_a^b \left(R^2 - r^2 \right) dx$
Shell $V = 2\pi \int_a^b rh \, dx$	**Cross Section** $V = \int_a^b A \, dx$