Derivatives, Integrals, and Properties
Of Inverse Trigonometric Functions and Hyperbolic Functions

(On this handout, a represents a constant, u and x represent variable quantities)

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad (|u|<1)$$

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad (|u|<1)$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad (|u|>1)$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad (|u|>1)$$

$$\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx}$$

Integrals Involving Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad (\text{Valid for } u^2 < a^2)$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad (\text{Valid for all } u)$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad (\text{Valid for } u^2 > a^2)$$

Derivatives, Integrals, and Properties
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The Six Basic Hyperbolic Functions
$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$
$\text{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$
$\text{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$
$\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Identities for Hyperbolic Functions

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$

$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

Derivatives of Hyperbolic Functions
$$\frac{d}{dx}\sinh u = \cosh u \frac{du}{dx}$$ $$\frac{d}{dx}\cosh u = \sinh u \frac{du}{dx}$$ $$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx}$$ $$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$$ $$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$$ $$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$$

Integrals of Hyperbolic Functions
$\int \sinh u \, du = \cosh u + C$
$\int \cosh u \, du = \sinh u + C$
$\int \operatorname{sech}^2 u \, du = \tanh u + C$
$\int \operatorname{csch}^2 u \, du = -\coth u + C$
$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$
$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

Inverse Hyperbolic Identities
$\operatorname{sech}^{-1} x = \cosh^{-1}\left(\frac{1}{x}\right)$
$\operatorname{csch}^{-1} x = \sinh^{-1}\left(\frac{1}{x}\right)$
$\coth^{-1} x = \tanh^{-1}\left(\frac{1}{x}\right)$

Derivatives of Inverse Hyperbolic Functions	
$\frac{d}{dx}\sinh^{-1} u = \frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$	
$\frac{d}{dx}\cosh^{-1} u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$	$(u > 1)$
$\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$	$(\lvert u\rvert < 1)$
$\frac{d}{dx}\operatorname{csch}^{-1} u = \frac{-1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}$	$(u \neq 0)$
$\frac{d}{dx}\operatorname{sech}^{-1} u = \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx}$	$(0 < u < 1)$
$\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2}\frac{du}{dx}$	$(\lvert u\rvert > 1)$

Integrals Involving Inverse Hyperbolic Functions

$$\int \frac{1}{\sqrt{a^2+u^2}}\,du = \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad (a > 0)$$

$$\int \frac{1}{\sqrt{u^2-a^2}}\,du = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad (u > a > 0)$$

$$\int \frac{1}{a^2-u^2}\,du = \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C & (\text{if } u^2 < a^2) \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C & (\text{if } u^2 > a^2) \end{cases}$$

$$\int \frac{1}{u\sqrt{a^2-u^2}}\,du = -\frac{1}{a}\,\mathrm{sech}^{-1}\left(\frac{u}{a}\right) + C \qquad (0 < u < a)$$

$$\int \frac{1}{u\sqrt{a^2+u^2}}\,du = -\frac{1}{a}\,\mathrm{csch}^{-1}\left|\frac{u}{a}\right| + C$$

Expressing Inverse Hyperbolic Functions As Natural Logarithms

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}) \qquad (-\infty < x < \infty)$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2-1}) \qquad (x \geq 1)$$

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad (|x| < 1)$$

$$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \qquad (0 < x \quad 1)$$

$$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right) \qquad (x \neq 0)$$

$$\coth^{-1} x = \frac{1}{2}\ln\frac{x+1}{x-1} \qquad (|x| > 1)$$

Alternate Form For Integrals Involving Inverse Hyperbolic Functions

$$\int \frac{1}{\sqrt{u^2 \pm a^2}}\, du = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$\int \frac{1}{a^2 - u^2}\, du = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + C$$

$$\int \frac{1}{u\sqrt{a^2 \pm u^2}}\, du = -\frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 \pm u^2}}{|u|} \right) + C$$