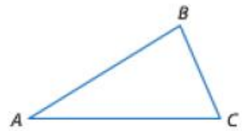
Classifying Triangles

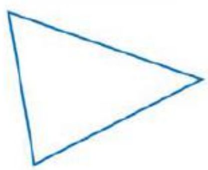


The sides of ΔABC are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
The vertices are points A, B, and C.
The angles are $\angle BAC$ or $\angle A$, $\angle ABC$ or $\angle B$, and $\angle BCA$ or $\angle C$.

Triangles can be classified in two ways – by their ***angles*** or by their ***sides***.

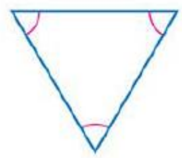
KeyConcept Classifications of Triangles by Angles

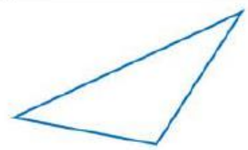
acute triangle



3 acute angles

equiangular triangle





1 obtuse angle

right triangle

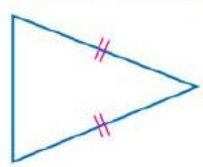
1 right angle

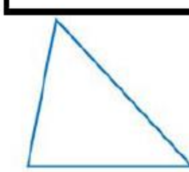
KeyConcept Classifications of Triangles by Sides

equilateral triangle

3 congruent sides

isosceles triangle





no congruent sides

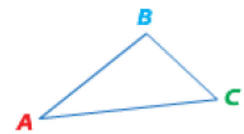
Section 4.2 Notes: Angles of Triangles

The Triangle Angle-Sum Theorem can be used to determine the measure of the third angle of a triangle when the other two angle measures are known.

Theorem 4.1

Words The sum of the measures of the angles of a triangle is 180.

Example $m\angle A + m\angle B + m\angle C = 180$

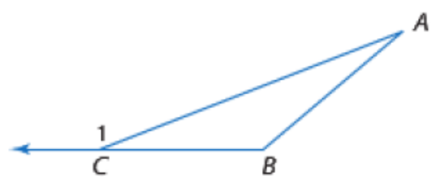


Auxiliary line: an extra line or segment drawn in a figure to help analyze geometry relationships.

Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

Example $m\angle A + m\angle B = m\angle 1$

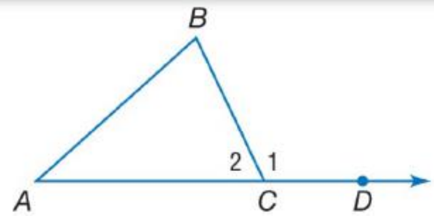


A **flow proof** uses statements written in boxes and arrows to show the logical progression of an argument. The reason justifying each statement is written below the box. See below for an example.

Proof Exterior Angle Theorem

Given: $\triangle ABC$

Prove: $m\angle A + m\angle B = m\angle 1$



Flow Proof:

$\triangle ABC$
Given
↓

$\angle 2$ and $\angle 1$ form a linear pair.
Definition of a linear pair
↓
$\angle 2$ and $\angle 1$ are supplementary.
If 2 $\angle$s form a linear pair, they are supplementary.
↓
$m\angle 2 + m\angle 1 = 180$
Definition of supplementary

$m\angle A + m\angle B + m\angle 2 = 180$
Triangle Angle-Sum Theorem

↘ ↙

$m\angle A + m\angle B + m\angle 2 = m\angle 2 + m\angle 1$
Substitution
↓
$m\angle A + m\angle B = m\angle 1$
Subtraction Property of Equality

Section 4.3 Notes: Congruent Triangles

If two geometric figures have exactly the same shape and size, they are **congruent**.

Congruent	Not Congruent
	
While positioned differently, Figures 1, 2, and 3 are exactly the same shape and size.	Figures 4 and 5 are exactly the same shape but not the same size. Figures 5 and 6 are the same size but not exactly the same shape.

In two **congruent polygons**, all of the parts of one polygon are congruent to the **corresponding parts** or matching parts of the other polygon. These corresponding parts include *corresponding angles* and *corresponding sides*.

KeyConcept Definition of Congruent Polygons

Words Two polygons are congruent if and only if their ______________ are congruent.

Example

Corresponding Angles

$\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$

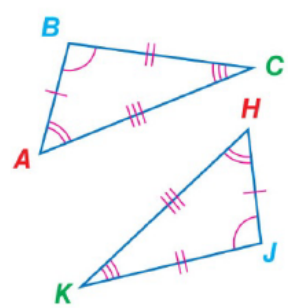
Corresponding Sides

$\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$

Congruence Statement

$\triangle ABC \cong \triangle HJK$

Model



Other congruence statements for the triangles above exist. Valid congruence statements for congruent polygons list corresponding vertices in the same order.

Valid Statement

$\triangle BCA \cong \triangle JKH$

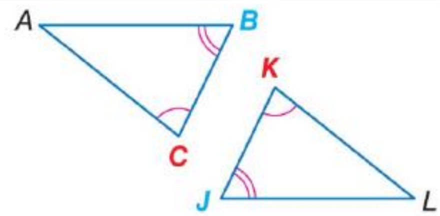
Not a Valid Statement

$\triangle ABC \cong \triangle HKJ$

Theorem

Words: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example: If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.



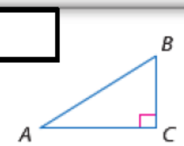
A **corollary** is a theorem with a proof that follows as a direct result of another theorem.

Corollaries Triangle Angle-Sum Corollaries

4.1

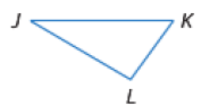
Abbreviation: *Acute ∠s of a rt. △ are comp.*

Example: If $\angle C$ is a right angle, then $\angle A$ and $\angle B$ are complementary.



4.2

Example: If $\angle L$ is a right or an obtuse angle, then $\angle J$ and $\angle K$ must be acute angles.



Theorem 4.4 Properties of Triangle Congruence

Property of Triangle Congruence

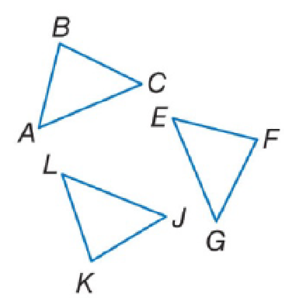
$\triangle ABC \cong \triangle ABC$

Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



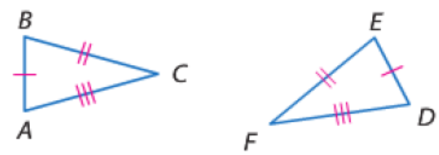
Section 4.4 Notes: Proving Triangles Congruent – SSS, SAS

In Lesson 4-3, you proved that two triangles were congruent by showing that all six pairs of corresponding parts were congruent. It is possible to prove two triangles congruent using fewer pairs.

Postulate 4.1

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.

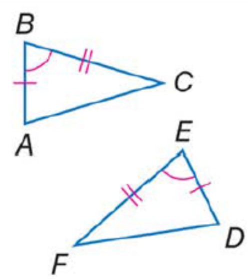


The angle formed by two adjacent sides of a polygon is called an **included angle**.

Postulate 4.2

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.



Things to Look for in a Δ Proof:

NAME PICTURE REASON TO USE

"Bow tie"

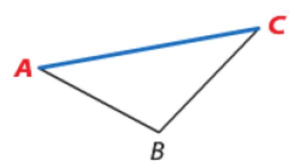
"Share a side"

Prove: $\overline{(2\text{ letters})} \cong \overline{(2\text{ diff. letters})}$

Prove: $\angle(3\text{ letters}) \cong \angle(3\text{ diff. letters})$

Geometry
Section 4.5 Notes: Proving Triangles Congruent – ASA, AAS

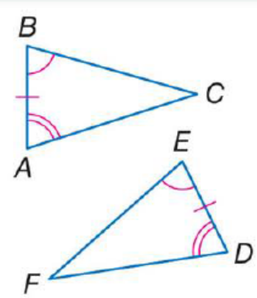
An **included side** is the side located between two consecutive angles of a polygon. In ΔABC, $\overline{AC}$ is the included side between ∠A and ∠C.



Postulate 4.3

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AB} \cong \overline{DE}$, and
Angle $\angle B \cong \angle E$,
then $\triangle ABC \cong \triangle DEF$.



Theorem

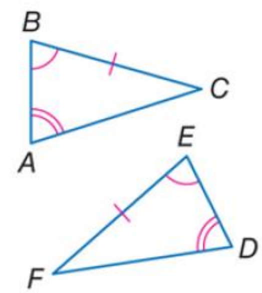
If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

Example If **A**ngle $\angle A \cong \angle D$,

Angle $\angle B \cong \angle E$, and

Side $\overline{BC} \cong \overline{EF}$,

then $\triangle ABC \cong \triangle DEF$.



ConceptSummary Proving Triangles Congruent

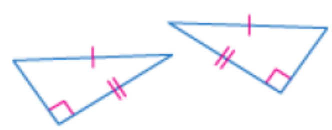
SSS	SAS	ASA	AAS
Three pairs of corresponding sides are congruent.	Two pairs of corresponding sides and their included angles are congruent.	Two pairs of corresponding angles and their included sides are congruent.	Two pairs of corresponding angles and the corresponding nonincluded sides are congruent.

Section 4.5 Extension: Proving RIGHT TRIANGLES Congruent

Theorem Right Triangle Congruence

Theorem 4.9 Hypotenuse-Leg Congruence
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Abbreviation *HL*

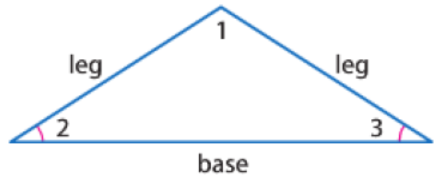


Section 4.6 Notes: Isosceles and Equilateral Triangles

The two congruent sides are called **the legs of an isosceles triangle**, and the angle with the sides that are the legs is called the **vertex angle**. The side of the triangle opposite the vertex angle is called the **base**. The two angles formed by the base and the congruent sides are called the **base angles**.

$\angle 1$ is the vertex angle.

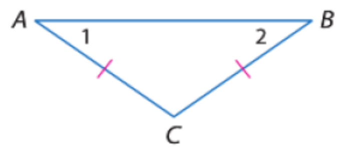
$\angle 2$ and $\angle 3$ are the base angles.



Theorems Isosceles Triangle

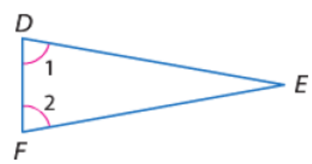
4.10 Isosceles Triangle Theorem

Example If $\overline{AC} \cong \overline{BC}$, then $\angle 2 \cong \angle 1$.



4.11 Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

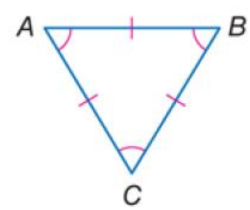


The Isosceles Triangle Theorem leads to two corollaries about the angles of an equilateral triangle.

Corollaries Equilateral Triangle

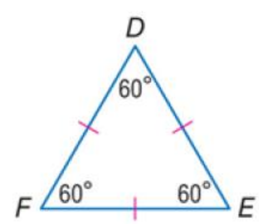
4.3 A triangle is equilateral if and only if it is equiangular.

Example If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.



4.4 Each angle of an equilateral triangle measures 60.

Example If $\overline{DE} \cong \overline{EF} \cong \overline{FE}$, then $m\angle A = m\angle B = m\angle C = 60$.



KeyConcept Placing Triangles on Coordinate Plane

Step 1 Use the origin as a vertex or center of the triangle.

Step 2 Place at least one side of a triangle on an axis.

Step 3 Keep the triangle within the first quadrant if possible.

Step 4 Use coordinates that make computations as simple as possible.