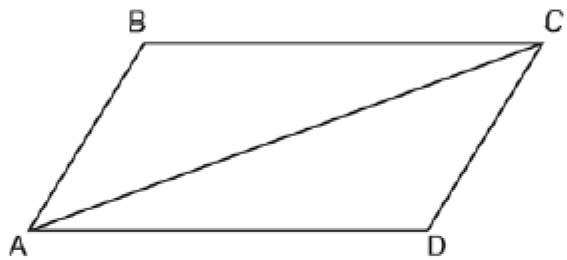
1 Given that $ABCD$ is a parallelogram, a student wrote the proof below to show that a pair of its opposite angles are congruent.



Statement	Reason
1. ABCD is a parallelogram.	1. Given
2. $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a parallelogram are congruent.
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Postulate of Congruency
4. $\triangle ABC \cong \triangle CDA$	4. Side-Side-Side
5. $\angle B \cong \angle D$	5. ______________________

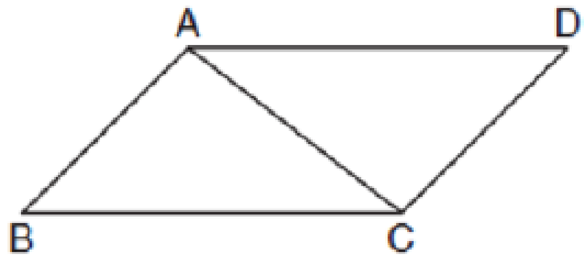
What is the reason justifying that $\angle B \cong \angle D$?

1) Opposite angles in a quadrilateral are congruent.
2) Parallel lines have congruent corresponding angles.
3) Corresponding parts of congruent triangles are congruent.
4) Alternate interior angles in congruent triangles are congruent.

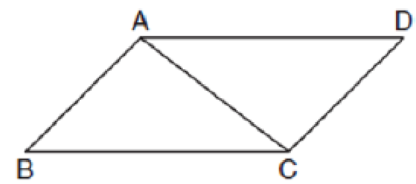
Answers

Statements	Reasons
	3 Corresponding parts of congruent triangles are congruent.

2 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



Prove: $\triangle ABC \cong \triangle CDA$

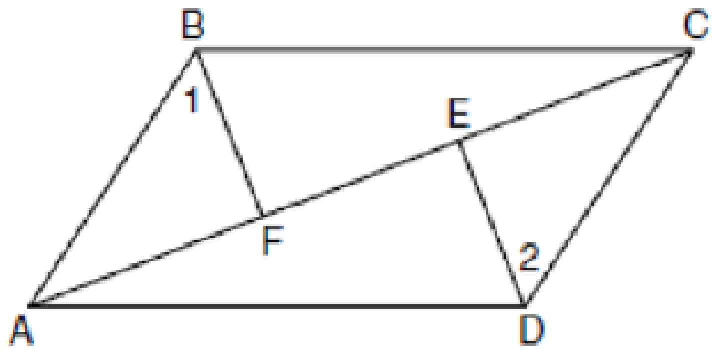


Answers

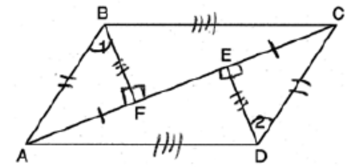
Statements	Reasons
Parallelogram *ABCD* with diagonal $\overline{AC}$ drawn	(given).
$\overline{AC} \cong \overline{AC}$	(reflexive property).
$\overline{AD} \cong \overline{CB}$ and $\overline{BA} \cong \overline{DC}$	(opposite sides of a parallelogram are congruent)
$\triangle ABC \cong \triangle CDA$	(SSS).

3 Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$

Prove: $ABCD$ is a parallelogram.



Answers



Statements	Reasons
$\overline{FE} \cong \overline{FE}$ (Reflexive Property)	
$\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{EF}$ (Line Segment Subtraction Theorem)	
$\overline{AF} \cong \overline{CE}$ (Substitution); $\angle BFA \cong \angle DEC$ (All right angles are congruent)	
$\triangle BFA \cong \triangle DEC$ (AAS);	

$\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$ (CPCTC);

$\angle BFC \cong \angle DEA$ (All right angles are congruent)

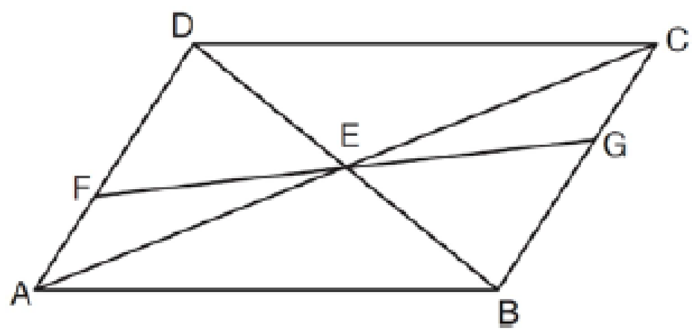
$\triangle BFC \cong \triangle DEA$ (SAS);

$\overline{AD} \cong \overline{CB}$ (CPCTC);

$ABCD$ is a parallelogram (opposite sides of quadrilateral $ABCD$ are congruent)

4

In the diagram below of quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$. Line segments AC, DB, and FG intersect at E. Prove: $\triangle AEF \cong \triangle CEG$



Answers

Statements	Reasons

Quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$ are given.

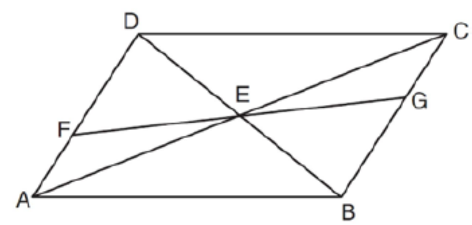
$\overline{AD} \parallel \overline{BC}$ because if two lines are cut by a transversal so that a pair of alternate interior angles are congruent, the lines are parallel.

$ABCD$ is a parallelogram because if one pair of opposite sides of a quadrilateral are both congruent and parallel, the quadrilateral is a parallelogram.

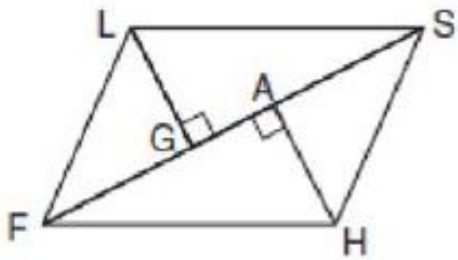
$\overline{AE} \cong \overline{CE}$ because the diagonals of a parallelogram bisect each other.

$\angle FEA \cong \angle GEC$ as vertical angles.

$\triangle AEF \cong \triangle CEG$ by ASA.



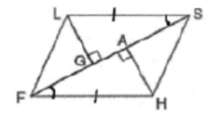
5 Given: parallelogram *FLSH*, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$



Prove: $\triangle LGS \cong \triangle HAF$

Answers

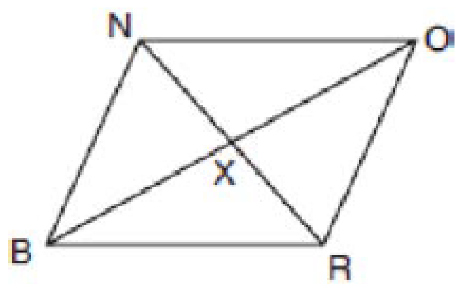
Statements	Reasons
$\overline{FH} \cong \overline{SL}$.	Because $FLSH$ is a parallelogram,
$\overline{FH} \parallel \overline{SL}$	Because $FLSH$ is a parallelogram,
$\angle AFH$ and $\angle LSG$ are alternate interior angles and congruent.	since $\overline{FGAS}$ is a transversal,



Therefore $\triangle LGS \cong \triangle HAF$ by AAS.

6

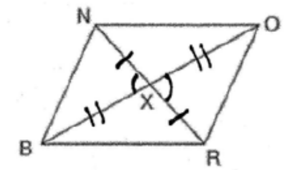
The accompanying diagram shows quadrilateral *BRON*, with diagonals $\overline{NR}$ and $\overline{BO}$, which bisect each other at *X*.



Prove: $\triangle BNX \cong \triangle ORX$

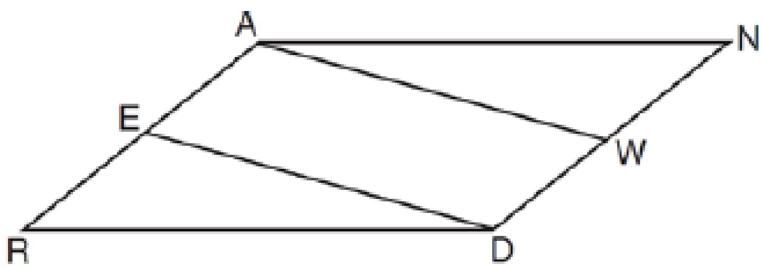
Answers

Statements	Reasons
$\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$.	Because diagonals $\overline{NR}$ and $\overline{BO}$ bisect each other,
$\angle BXN$ and $\angle OXR$ are congruent	vertical angles.
Therefore $\triangle BNX \cong \triangle ORX$ by SAS.	



7

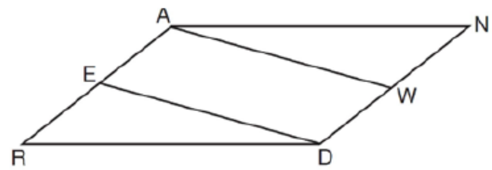
Given: Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points W and E, respectively



Prove that $\triangle ANW \cong \triangle DRE$. Prove that quadrilateral $AWDE$ is a parallelogram.

Answers

Statements	Reasons



Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points W and E (Given).

$\overline{AN} \cong \overline{RD}$, $\overline{AR} \cong \overline{DN}$ (Opposite sides of a parallelogram are congruent).

$AE = \frac{1}{2}AR$, $WD = \frac{1}{2}DN$, so $\overline{AE} \cong \overline{WD}$ (Definition of bisect and division property of equality).

$\overline{AR} \parallel \overline{DN}$ (Opposite sides of a parallelogram are parallel).

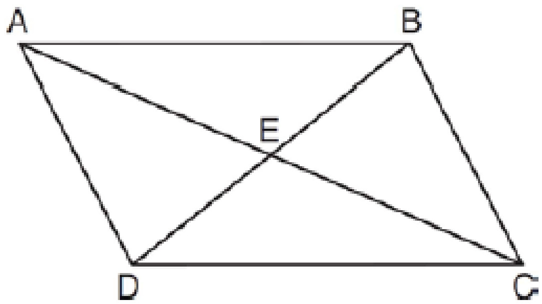
$AWDE$ is a parallelogram (Definition of parallelogram).

$RE = \frac{1}{2}AR$, $NW = \frac{1}{2}DN$, so $\overline{RE} \cong \overline{NW}$ (Definition of bisect and division property of equality).

$\overline{ED} \cong \overline{AW}$ (Opposite sides of a parallelogram are congruent). $\triangle ANW \cong \triangle DRE$ (SSS).

8

Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E

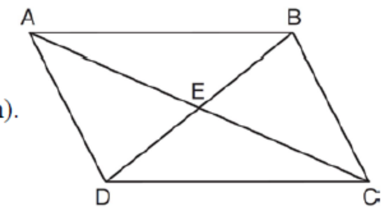


Prove: $\triangle AED \cong \triangle CEB$

Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

Answers

Statements	Reasons



Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E (Given).

$\overline{AD} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent).

$\angle AED \cong \angle CEB$ (Vertical angles are congruent).

$\overline{BC} \parallel \overline{DA}$ (Definition of parallelogram).

$\angle DBC \cong \angle BDA$ (Alternate interior angles are congruent).

$\triangle AED \cong \triangle CEB$ (AAS). 180° rotation of $\triangle AED$ around point E.