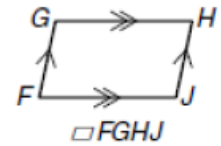# Parallelogram
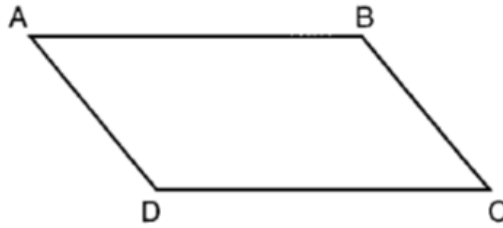
A parallelogram is a quadrilateral with two pairs of parallel sides. All parallelograms, such as $\square FGHJ$, have the following properties.

$\square FGHJ$

Properties of Parallelograms			
Opposite sides are congruent.	$\overline{FG} \cong \overline{HJ}$ $\overline{GH} \cong \overline{JF}$	Opposite angles are congruent.	$\angle F \cong \angle H$ $\angle G \cong \angle J$
Consecutive angles are supplementary.	$m\angle F + m\angle G = 180°$ $m\angle G + m\angle H = 180°$ $m\angle H + m\angle J = 180°$ $m\angle J + m\angle F = 180°$	The diagonals bisect each other.	$\overline{FP} \cong \overline{HP}$ $\overline{GP} \cong \overline{JP}$

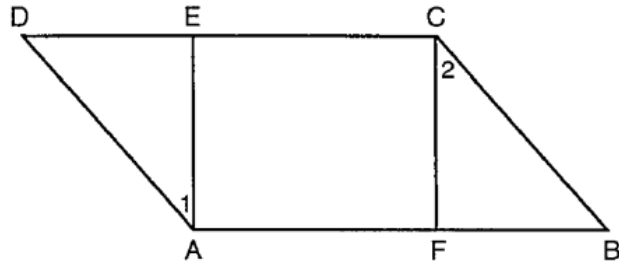Complete the statement and give the reason that justifies the statement.

Given: ABCD is a parallelogram

A _____ B

D _____ C

Statements	Reasons
a. $\overline{AB} \cong$ _____ and $\overline{AD} \cong$ _____	a.
b. $\measuredangle A \cong$ _____ and $\measuredangle D \cong$ _____	b.
c. $\overline{AB} \parallel$ _____ and $\overline{AD} \parallel$ _____	c.
d. $\measuredangle A$ *suppl.* _____ and _____ $\measuredangle C$ *suppl.* _____ and _____	d.
e. Draw $\overline{AC}$ and $\overline{BD}$. (The lines intersect at E.)	e. Two Points Make a Line.
f. $\measuredangle BAC \cong$ _____ and $\measuredangle DAC \cong$ _____	f.
g. $\overline{AE} \cong$ _____ and $\overline{DE} \cong$ _____	g.

Given: ▱ ABCD
$\overline{DE} \cong \overline{FB}$

Prove: a) △DEA ≅ △BFC
       b) ∠1 ≅ ∠2

STATEMENT	REASONS
1. Parallelogram ABCD	1. Given
2. $\overline{AD} \cong$ _____	2.
3. $\angle D \cong$ _____	3.
4. $\overline{DE} \cong \overline{FB}$	4. Given
5.	5.
6.	6.

Given: ⟡ ABCD

Prove: △AEB ≅ △CED
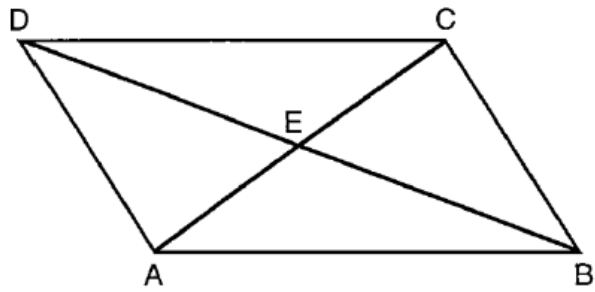
STATEMENT	REASONS
1. Parallelogram ABCD	1. Given
2. $\overline{AB}$ ≅ _____	2.
3. $\overline{AB}$ ‖ _____	3.
4. ∠CAB ≅ _____	4.
5. ∠AEB ≅ _____	5.
6.	6.

Given: $\square ABCD$

Prove: $\triangle DAC \cong \triangle BCA$

(At most 6 steps. You may not need all 6!)
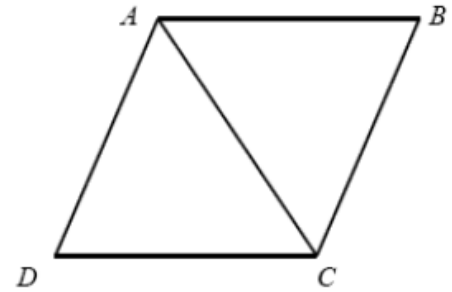
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6

Given: ▱ GHIJ
$\overline{HP} \cong \overline{JQ}$

Prove: $\overline{PX} \cong \overline{QX}$
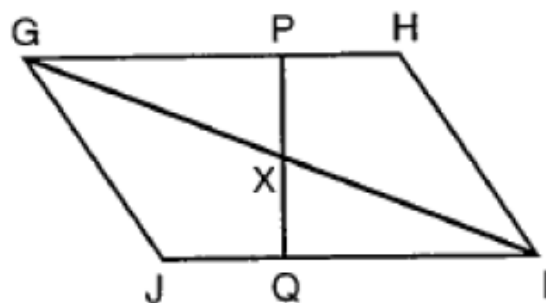
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

Given: *WOSB* is a parallelogram
$\angle 3 \cong \angle 4$
$\overline{MW} \cong \overline{SN}$

Prove: $\overline{IM} \cong \overline{LN}$
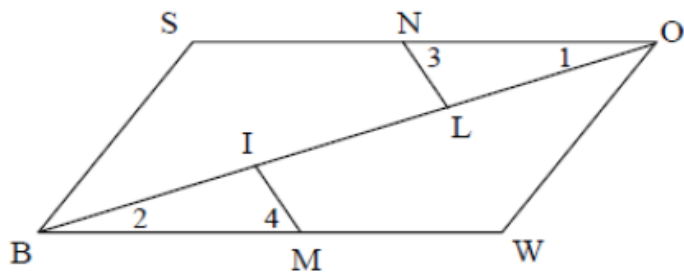
Statements	Reasons

Challenge

Given: $ABCD$ is a parallelogram.
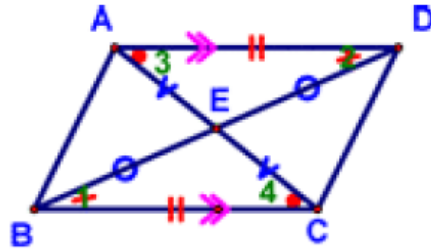$\overline{DE} \perp \overline{AF}$. $\overline{CF} \perp \overline{AF}$.

Prove: $\overline{DE} \cong \overline{CF}$.

Given: Parallelogram ABCD

**Prove:** $\overline{AC}$ and $\overline{DB}$ bisect each other

Statements	Reasons
1. Parallelogram ABCD	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. Opposite sides of a parallelogram are $\parallel$
A 3. $\angle 1 \cong \angle 2$	3. $\parallel \rightarrow$ *AIA's* $\cong$
A 4. $\angle 3 \cong \angle 4$	4. $\parallel \rightarrow$ *AIA's* $\cong$
S 5. $\overline{AD} \cong \overline{BC}$	5. Opposite sides of a parallelogram are $\cong$
6. $\triangle BEC \cong \triangle DEA$	6. ASA (3, 5, 4)
7. $\overline{DE} \cong \overline{BE}$	7. CPCTC
8. $\overline{AE} \cong \overline{CE}$	8. CPCTC
9. $\overline{AC}$ and $\overline{DB}$ bisect each other	9. 2 $\cong$ segs $\rightarrow$ *segment bisector*

Identify the reason that proves $\triangle ABD \cong \triangle CDB$.

A) SSS

B) SAS

C) ASA

D) AAS
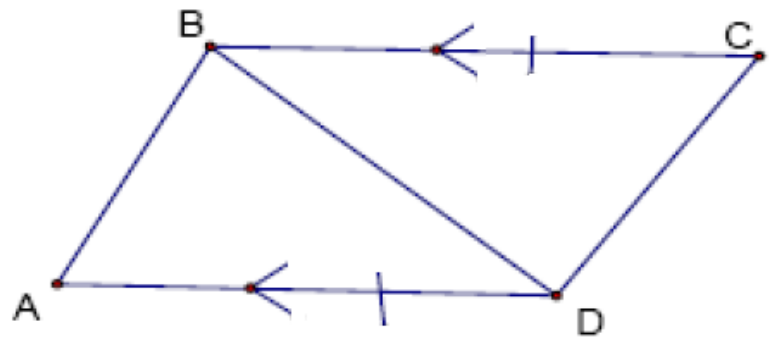
Given: ABCD is a parallelogram.
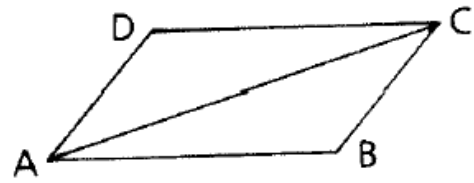
Prove: $\triangle AEB \cong \triangle CED$
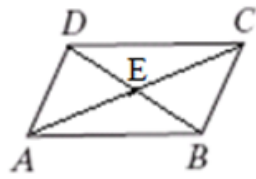
Statements	Reasons

Given: ▱ ABCD (ABCD is a ▱)
Conclusion: △ABC ≅ △CDA

Statements	Reasons

Given: ABCD is a ▱ (parallelogram).
   ∠GHA ≅ ∠FEC,
   $\overline{HB} \cong \overline{DE}$

Conclusion: $\overline{GH} \cong \overline{EF}$
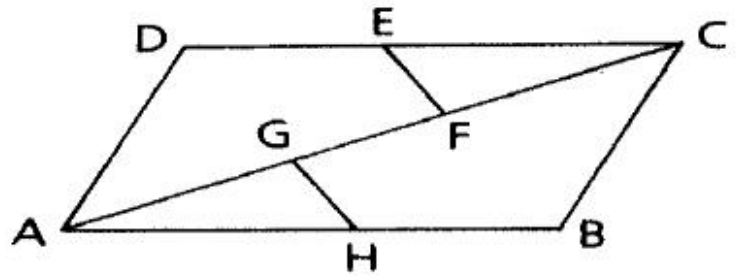
Statement	Reason
1	1 Given
2	2 Opposite sides of a ▱ are ∥.
3	3 ∥ lines ⇒ alt. int. ∠s ≅
4	4 Opposite sides of a ▱ are ≅.
5	5 Given
6	6 Subtraction Property (___ . ___)
7	7 Given
8	8 ASA (3, 6, 7)
9	9 CPCTC

Given: □ EFHJ,
∠1 ≅ ∠2
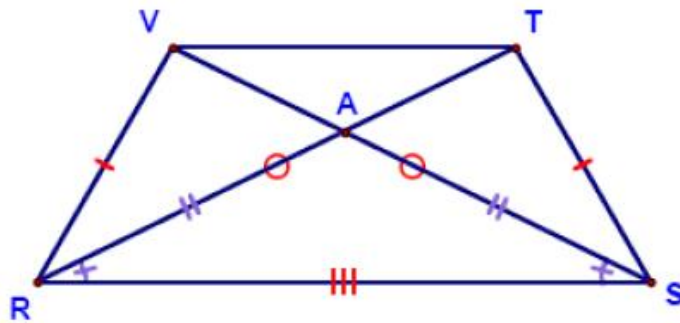Conclusion: $\overline{KH} \cong \overline{EG}$
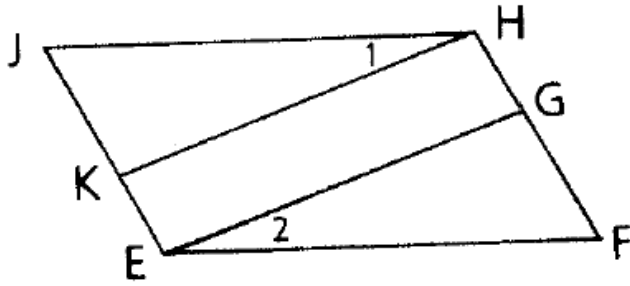
Statements	Reasons

Isosceles trapezoid $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. If $AC = 5x + 13$ and $BD = 11x - 5$, what is the value of x?

1) 28

2) $10\dfrac{3}{4}$

3) 3

4) $\dfrac{1}{2}$

Given: RVTS is an isos. trapezoid with legs $\overline{VR}$ & $\overline{TS}$

Prove: $\triangle ARS$ is isosceles

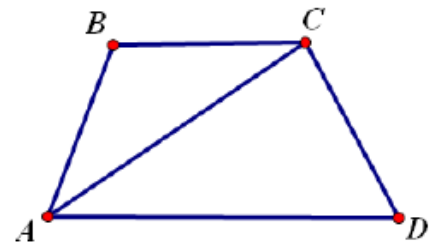| Statements	Reasons
1. RVTS is an isos. trapezoid with legs $\overline{VR}$ & $\overline{TS}$	1. Given
S 2. $\overline{VR} \cong \overline{TS}$	2. Legs of an Isos. Trap are $\cong$
S 3. $\overline{VS} \cong \overline{TR}$	3. Diagonals of an Isos. Trap are $\cong$
S 4. $\overline{RS} \cong \overline{RS}$	4. Reflexive Property
5. $\triangle VRS \cong \triangle TSR$	5. SSS (2, 3, 4)
6. $\angle TRS \cong \angle VSR$	6. CPCTC
7. $\overline{AR} \cong \overline{AS}$	7. If $\triangle$, then $\triangle$
8. $\triangle ARS$ is isosceles	8. If $\triangle$, then Isos. $\triangle$

Challenge:

Given: $\overrightarrow{AC}$ bisects $\sphericalangle BAD$.
$\overline{AB} \cong \overline{BC}$,
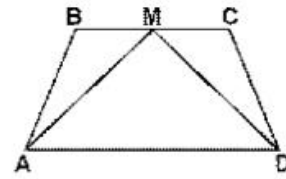$\overline{AB} \nparallel \overline{CD}$

Prove: ABCD is a trapezoid.

Given: ABCD is an isosceles trapezoid

$\overline{AD} \parallel \overline{BC}$

M is the midpoint of $\overline{BC}$

Prove: ΔMAD is isosceles

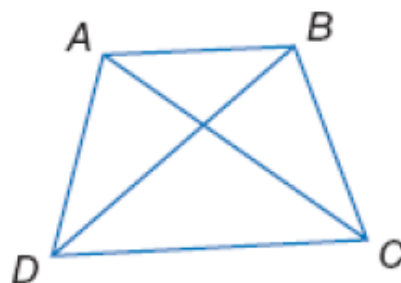| Statements	Reasons

Given: *ABCD* is an isosceles trapezoid.

Prove: $\triangle ADC \cong \triangle BCD$

Statements	Reasons

Given: Isosceles Trapezoid RQPO ($\overline{OR}$ and $\overline{PQ}$ are bases.)
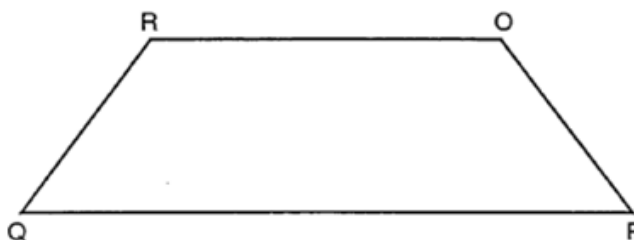
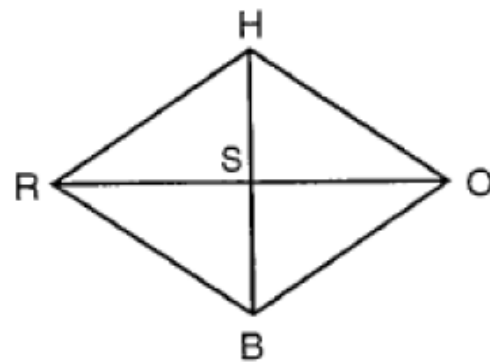| STATEMENT	REASONS
a. $\overline{RQ} \parallel$ _____	a.
b. $\overline{RO} \cong$ _____	b.
c. $\angle O \cong$ _____	c.
d. $\angle Q \cong$ _____	d.
e. Draw $\overline{QO}$ and $\overline{RP}$	e. 2 points make a line
f. $\overline{QO} \cong$ _____	f.
g. $\angle QRP \cong$ _____	g.
h. $\angle RQO \cong$ _____	h.

Given: Trapezoid TRPA ($\overline{AP}$ and $\overline{TR}$ are bases)
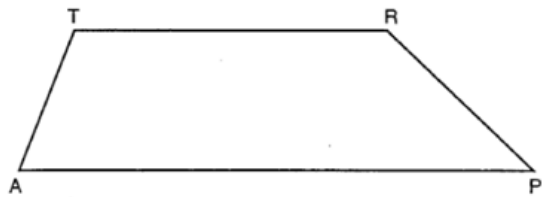
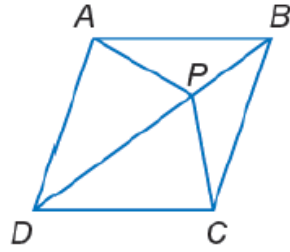| STATEMENT	REASONS
a. $\overline{TR} \parallel$ _____	a.
b. $\angle T$ and _____ are supplementary	b.
c. $\angle R$ and _____ are supplementary	c.

Given: $\overline{HS} \cong \overline{SB}$, $\overline{RS} \cong \overline{SO}$
$\overline{HR} \cong \overline{HO}$

Prove: RHOB is a rhombus

Statements	Reasons
1.	1. Given
2. Δ ____ ≅ Δ ____	2. (__ , __ , __)
3. RHOB is a Parallelogram	3. _____ of a quadrilateral bisect each other → _____
4. RHOB is a Rhombus	4. ≅ _____ sides of a Parallelogram → Rhombus

Write a two-column proof to prove that if $ABCD$ is a rhombus with diagonal $\overline{DB}$, then $\overline{AP} \cong \overline{CP}$.

Statements	Reasons

Given: TBCD is a rhombus
∠RTB ≅ ∠FCB

Prove: $\overline{RD} \cong \overline{DF}$
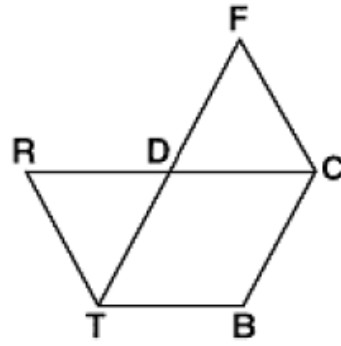
Statements	Reasons

Given: Rectangle WXYZ, M is the midpoint of $\overline{WX}$.

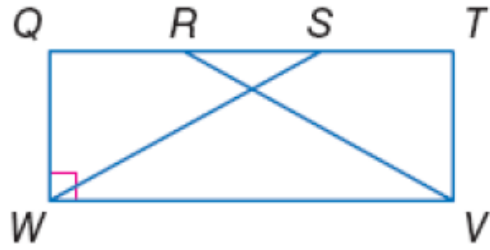Prove: $\triangle$ ZMY is isosceles.
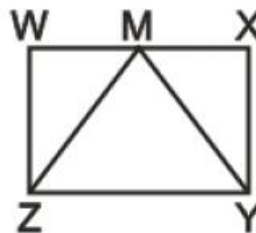
Statements	Reasons

Given: $QTVW$ is a rectangle.

$\overline{QR} \cong \overline{ST}$

Prove: $\triangle SWQ \cong \triangle RVT$

Statements	Reasons

Find the value of each variable in the rhombus.

a = _____

b = _____

c = _____

d = _____

Given: ABCD is a rhombus.
        diagonals $\overline{AC}$, $\overline{BD}$

Prove: △AED ≅ △AEB ≅
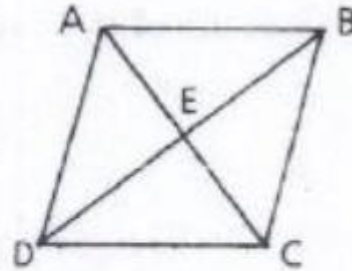   △BEC ≅ △CED

1 ABCD is a rhombus.   1 Given

2 Diagonals $\overline{AC}$, $\overline{BD}$ 2 Given

3 $\overline{AB}$ ≅ $\overline{BC}$ ≅ $\overline{CD}$ ≅ $\overline{DA}$ 3 All sides of a rhombus ≅.

4 $\overline{AC}$, $\overline{BD}$ ⊥ bis each other.   4 Diag of rhombus ⊥ bis
                                                   each other.

5 $\overline{AE}$ ≅ $\overline{EC}$, $\overline{DE}$ ≅ $\overline{EB}$ 5 Bis divides a seg into 2 ≅
                                                   segs.

6 △AED ≅ △AEB ≅   6 SSS
   △BEC ≅ △CED

Given: ABCD is a rhombus.

diagonals $\overline{AC}$, $\overline{BD}$

Prove: △AED ≅ △AEB ≅

△BEC ≅ △CED

1 ABCD is a rhombus.	1 Given
2 Diagonals $\overline{AC}$, $\overline{BD}$	2 Given
3 $\overline{AB}$ ≅ $\overline{BC}$ ≅ $\overline{CD}$ ≅ $\overline{DA}$	3 All sides of a rhombus ≅.
4 $\overline{AC}$, $\overline{BD}$ ⊥ bis each other.	4 Diag of rhombus ⊥ bis each other.
5 $\overline{AE}$ ≅ $\overline{EC}$, $\overline{DE}$ ≅ $\overline{EB}$	5 Bis divides a seg into 2 ≅ segs.
6 △AED ≅ △AEB ≅ △BEC ≅ △CED	6 SSS

Given: ☐ MPRS

$\overline{MO} \cong \overline{PO}$

Prove: △ROS is isos.
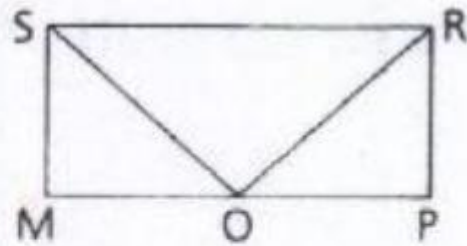
1 ☐ MPRS	1 Given
2 $\overline{MO} \cong \overline{PO}$	2 Given
3 $\overline{SM} \cong \overline{RP}$	3 Opp sides ≅ in a ☐.
4 ∠M is a rt ∠.	4 In a ☐, all ∠s are rt ∠s.
5 ∠P is a rt ∠.	5 Same as 4
6 ∠M ≅ ∠P	6 All rt ∠s are ≅.
7 △SMO ≅ △RPO	7 SAS
8 $\overline{SO} \cong \overline{RO}$	8 CPCTC
9 △ROS is isos.	9 An isos △ has 2 sides ≅.

Challenge

Given: YTWX is a ▱
$\overline{YP} \perp \overline{TW}$
$\overline{ZW} \perp \overline{TY}$
$\overline{TP} \cong \overline{TZ}$
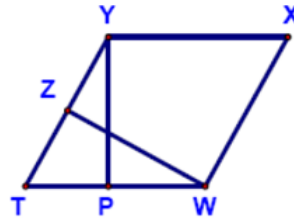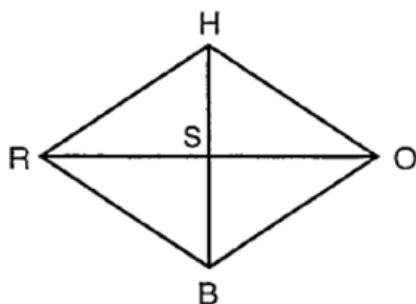
Prove: YTWX is a rhombus

Statements	Reasons

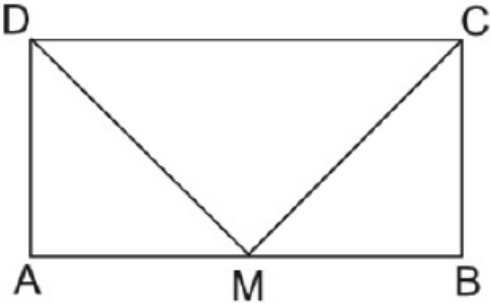Given: Rhombus RHOB

Prove: ∠HSR ≅ ∠HSO

Statements	Reasons

Given: $ABCD$ is a rectangle

M is midpoint of $\overline{AB}$
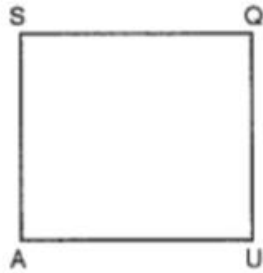
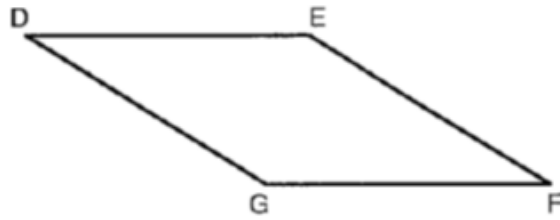**Prove:** $\overline{DM} \cong \overline{CM}$

Statements	Reasons

Given: Square SQUA

S ——— Q

A ——— U
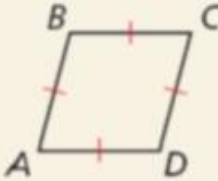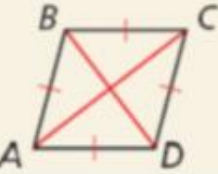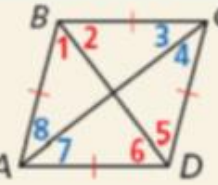
STATEMENT	REASONS
a. ∠S is a _____ angle	a.
b. $\overline{SQ} \cong \overline{QU} \cong \overline{UA} \cong \overline{SA}$	b.
c. ∠S ≅ ∠Q ≅ ∠U ≅ ∠A	c.
d. Draw $\overline{SU}$ and $\overline{QA}$	d. 2 points make a line
e. $\overline{SU}$ ⊥ _____	e.
f. $\overline{SU}$ ≅ _____	f.
g. ∠ASU ≅ ∠QSU	g.
h. ∠SUA ≅ ∠QUS	h.

Given: Rhombus DEFG

| STATEMENT	REASONS
a. $\overline{DE} \cong \overline{EF} \cong \overline{FG} \cong \overline{GD}$	a.
b. $\angle D \cong \angle F, \quad \angle E \cong \angle G$	b.
c. Draw $\overline{DF}$	c. 2 points make a line
d. $\angle GDF \cong \angle EDF$	d.
e. $\angle EFD \cong \angle GFD$	e.
f. Draw $\overline{EG}$ (to intersect $\overline{DF}$ at X)	f. 2 points make a line
g. $\overline{DF} \perp$ _____	g.
h. $\overline{EX} \cong$ _____ and $\overline{DX} \cong$ _____	h.

..... Rhombi

Theorems (**Properties of Rhombuses**)

THEOREM	HYPOTHESIS	CONCLUSION
6-4-3 If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus → ▱)		ABCD is a parallelogram.
6-4-4 If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus → diags. ⊥)		$\overline{AC} \perp \overline{BD}$
6-4-5 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus → each diag. bisects opp. ∢)		∠1 ≅ ∠2 ∠3 ≅ ∠4 ∠5 ≅ ∠6 ∠7 ≅ ∠8

Theorems — **Properties of Rectangles**

THEOREM	HYPOTHESIS	CONCLUSION
6-4-1 If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → ▱)	B ⬚ C / A ⬚ D	$ABCD$ is a parallelogram.
6-4-2 If a parallelogram is a rectangle, then its diagonals are congruent. $\left(\text{rect.} \rightarrow \text{diags.} \cong\right)$	B ⊠ C / A ⊠ D	$\overline{AC} \cong \overline{BD}$

Properties of Parallelograms
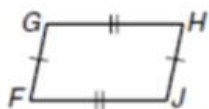
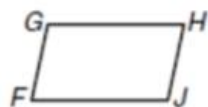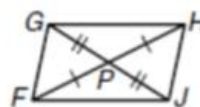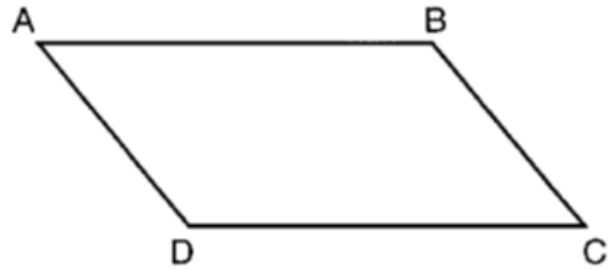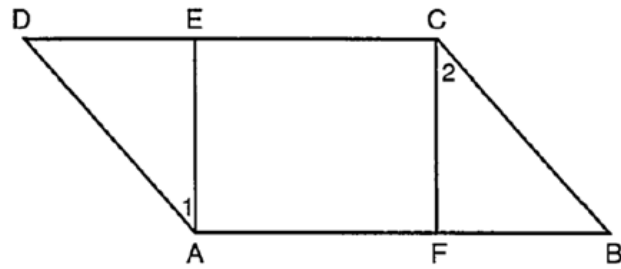|	
$\overline{FG} \cong \overline{HJ}$ $\overline{GH} \cong \overline{JF}$ Opposite sides are congruent.	$\angle F \cong \angle H$ $\angle G \cong \angle J$ Opposite angles are congruent.
$m\angle F + m\angle G = 180°$ $m\angle G + m\angle H = 180°$ $m\angle H + m\angle J = 180°$ $m\angle J + m\angle F = 180°$ Consecutive angles are supplementary.	$\overline{FP} \cong \overline{HP}$ $\overline{GP} \cong \overline{JP}$ The diagonals bisect each other.

Given: ABCD is a parallelogram

Statements	Reasons
a. $\overline{AB} \cong$ _____ and $\overline{AD} \cong$ _____	a.
b. ∢A $\cong$ _____ and ∢D $\cong$ _____	b.
c. $\overline{AB} \parallel$ _____ and $\overline{AD} \parallel$ _____	c.
d. ∢A *suppl.* _____ and _____ ∢C *suppl.* _____ and _____	d.
e. Draw $\overline{AC}$ and $\overline{BD}$. (The lines intersect at E.)	e. Two Points Make a Line.
f. ∢BAC $\cong$ _____ and ∢DAC $\cong$ _____	f.
g. $\overline{AE} \cong$ _____ and $\overline{DE} \cong$ _____	g.

Given: ▱ ABCD
$\overline{DE} \cong \overline{FB}$

Prove: a) △DEA ≅ △BFC
      b) ∠1 ≅ ∠2

STATEMENT	REASONS
Parallelogram ABCD	1. Given
$\overline{AD} \cong$ _____	2.
∠D ≅ _____	3.
$\overline{DE} \cong \overline{FB}$	4. Given
	5.
	6.