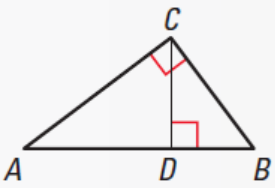
THEOREM

THEOREM 9.1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



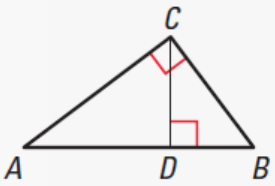
$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$

GEOMETRIC MEAN THEOREMS

THEOREM 9.2

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.



$$\frac{BD}{CD} = \frac{CD}{AD}$$

THEOREM 9.3

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

$$\frac{AB}{CB} = \frac{CB}{DB}$$

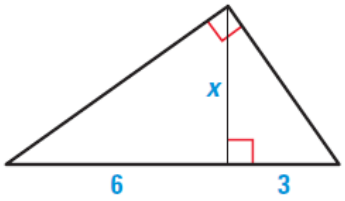
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$\frac{AB}{AC} = \frac{AC}{AD}$$

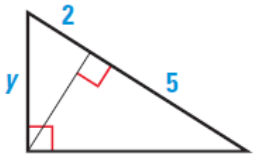
EXAMPLE 2 *Using a Geometric Mean*

Find the value of each variable.

a.



b.



SOLUTION

a. Apply Theorem 9.2.

$$\frac{6}{x} = \frac{x}{3}$$

$$18 = x^2$$

$$\sqrt{18} = x$$

$$\sqrt{9} \cdot \sqrt{2} = x$$

$$3\sqrt{2} = x$$

b. Apply Theorem 9.3.

$$\frac{5+2}{y} = \frac{y}{2}$$

$$\frac{7}{y} = \frac{y}{2}$$

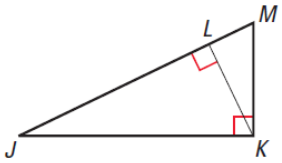
$$14 = y^2$$

$$\sqrt{14} = y$$

GUIDED PRACTICE

Vocabulary Check ✓

In Exercises 1–3, use the diagram at the right.



1. In the diagram, KL is the __?__ of ML and JL.

Concept Check ✓

2. Complete the following statement: $\triangle JKL \sim \triangle$ __?__ $\sim \triangle$ __?__ .

3. Which segment's length is the geometric mean of ML and MJ?

Skill Check ✓

In Exercises 4–9, use the diagram above. Complete the proportion.

4. $\frac{KM}{KL} = \frac{?}{JK}$

5. $\frac{JM}{?} = \frac{JK}{JL}$

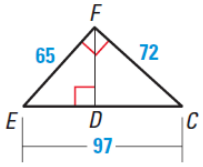
6. $\frac{?}{LK} = \frac{LK}{LM}$

7. $\frac{JM}{?} = \frac{KM}{LM}$

8. $\frac{LK}{LM} = \frac{JK}{?}$

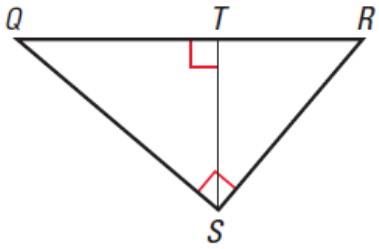
9. $\frac{?}{JK} = \frac{MK}{MJ}$

10. Use the diagram at the right. Find DC. Then find DF. Round decimals to the nearest tenth.



PRACTICE AND APPLICATIONS

SIMILAR TRIANGLES **Use the diagram.**

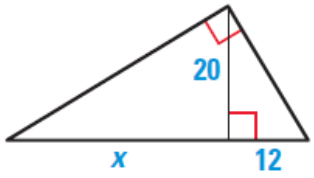


11. Sketch the three similar triangles in the diagram. Label the vertices.

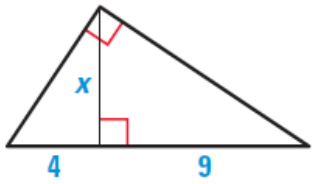
12. Write similarity statements for the three triangles.

USING PROPORTIONS **Complete and solve the proportion.**

13. $\frac{x}{20} = \frac{?}{12}$



14. $\frac{4}{x} = \frac{x}{?}$



15. $\frac{5}{x} = \frac{x}{?}$