

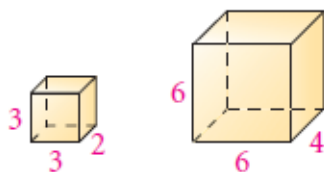
## Finding Relationships in Area and Volume

**Similar solids** have the same shape, and all their corresponding dimensions are proportional. The ratio of corresponding linear dimensions of two similar solids is the **similarity ratio**. Any two cubes are similar, as are any two spheres.

### 1 EXAMPLE Identifying Similar Solids

Are the two rectangular prisms similar? If so, give the similarity ratio.

a.

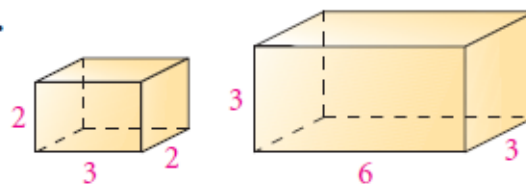


$$\frac{3}{6} = \frac{3}{6} = \frac{2}{4}$$

The rectangular prisms are similar because the ratios of the corresponding linear dimensions are equal.

The similarity ratio is  $\frac{1}{2}$ .

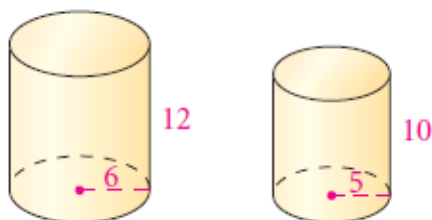
b.



$$\frac{2}{3} \neq \frac{3}{6}$$

The rectangular prisms are not similar because the ratios of corresponding linear dimensions are not equal.

- 1 Are the two cylinders similar?  
If so, give the similarity ratio.  
yes; 6 : 5



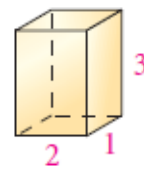
The two similar prisms shown here suggest two important relationships for similar solids.

The ratio of the side lengths is 1 : 2.

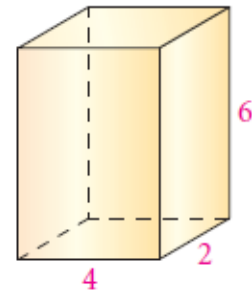
The ratio of the surface areas is 22 : 88, or 1 : 4.

The ratio of the volumes is 6 : 48, or 1 : 8.

The ratio of the surface areas equals the square of the similarity ratio. The ratio of the volumes equals the cube of the similarity ratio. These two facts apply to all similar solids.



$$\begin{aligned} \text{S.A.} &= 22 \text{ m}^2 \\ V &= 6 \text{ m}^3 \end{aligned}$$



$$\begin{aligned} \text{S.A.} &= 88 \text{ m}^2 \\ V &= 48 \text{ m}^3 \end{aligned}$$

### Theorem 11-12 Areas and Volumes of Similar Solids

If the similarity ratio of two similar solids is  $a : b$ , then

- (1) the ratio of their corresponding areas is  $a^2 : b^2$ , and
- (2) the ratio of their volumes is  $a^3 : b^3$ .

### EXAMPLE Finding the Similarity Ratio

Find the similarity ratio of two cubes with volumes of  $729 \text{ cm}^3$  and  $1331 \text{ cm}^3$ .

$$\frac{a^3}{b^3} = \frac{729}{1331} \quad \text{The ratio of the volumes is } a^3 : b^3.$$

$$\frac{a}{b} = \frac{9}{11} \quad \text{Take cube roots.}$$

- The similarity ratio is  $9 : 11$ .
- 2 Find the similarity ratio of two similar prisms with surface areas  $144 \text{ m}^2$  and  $324 \text{ m}^2$ .  
**2 : 3**

### EXAMPLE Using a Similarity Ratio

**Paint Cans** The lateral areas of two similar paint cans are  $1019 \text{ cm}^2$  and  $425 \text{ cm}^2$ . The volume of the small can is  $1157 \text{ cm}^3$ . Find the volume of the large can.



First find the similarity ratio  $a : b$ .

$$\frac{a^2}{b^2} = \frac{1019}{425} \quad \text{The ratio of the surface areas is } a^2 : b^2.$$

$$\frac{a}{b} = \frac{\sqrt{1019}}{\sqrt{425}} \quad \text{Take square roots.}$$

Use the similarity ratio to find the volume.

$$\frac{V_{\text{large}}}{V_{\text{small}}} = \frac{\sqrt{1019}^3}{\sqrt{425}^3} \quad \text{The ratio of the volumes is } a^3 : b^3.$$

$$\frac{V_{\text{large}}}{1157} = \frac{\sqrt{1019}^3}{\sqrt{425}^3} \quad \text{Substitute 1157 for } V_{\text{small}}.$$

$$V_{\text{large}} = 1157 \cdot \frac{\sqrt{1019}^3}{\sqrt{425}^3} \quad \text{Solve for } V_{\text{large}}.$$

$$V_{\text{large}} \approx 4295 \quad \text{Use a calculator.}$$

- The volume of the large paint can is about  $4295 \text{ cm}^3$ .

- 3 The volumes of two similar solids are  $128 \text{ m}^3$  and  $250 \text{ m}^3$ . The surface area of the larger solid is  $250 \text{ m}^2$ . What is the surface area of the smaller solid?  $160 \text{ m}^2$

### EXAMPLE

**Paperweights** A marble paperweight shaped like a pyramid weighs 0.15 lb. How much does a similarly shaped marble paperweight weigh if each dimension is three times as large?

The similarity ratio is  $1 : 3$ . The ratio of the volumes, and hence the ratio of the weights, is  $1^3 : 3^3$ , or  $1 : 27$ .

$$\frac{1}{27} = \frac{0.15}{x} \quad \text{Let } x = \text{the weight of the larger paperweight.}$$

$$x = 27(0.15) \quad \text{Use the Cross Product Property.}$$

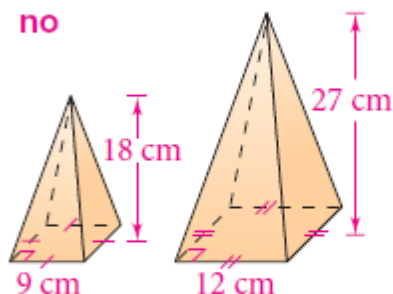
$$x = 4.05$$

- The larger paperweight weighs about 4 lb.

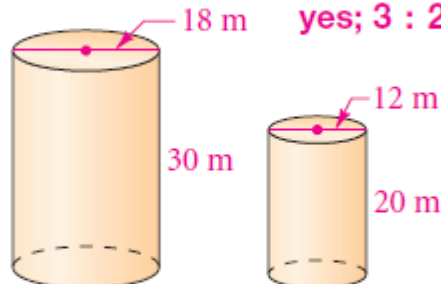
## Practice and Problem Solving

Are the two figures similar? If so, give the similarity ratio.

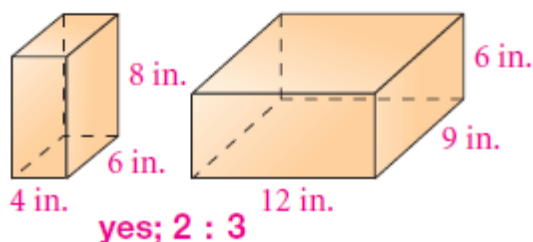
1. **no**



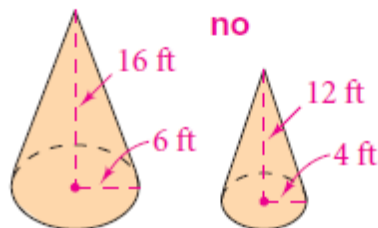
2. **yes; 3 : 2**



3.



4.

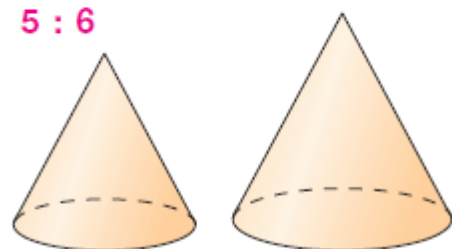


5. two cubes, one with 3-cm edges, the other with 4.5-cm edges **yes; 2 : 3**

6. a cylinder and a square prism each with 3-in. radii and 1-in. heights **no**

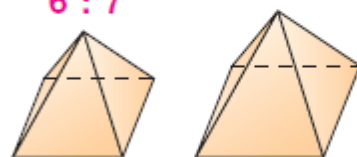
Each pair of figures is similar. Use the given information to find the similarity ratio of the smaller figure to the larger figure.

7. **5 : 6**



$$V = 250\pi \text{ ft}^3 \quad V = 432\pi \text{ ft}^3$$

8. **6 : 7**



$$V = 216 \text{ in.}^3 \quad V = 343 \text{ in.}^3$$

Copy and complete the table for the similar solids.

	Similarity Ratio	Ratio of Surface Areas	Ratio of Volumes	
30.	1 : 2	■ : ■	■ : ■	1 : 4; 1 : 8
31.	3 : 5	■ : ■	■ : ■	9 : 25; 27 : 125
32.	■ : ■	49 : 81	■ : ■	7 : 9; 343 : 729
33.	■ : ■	■ : ■	125 : 512	5 : 8; 25 : 64