Verifying Trigonometric Identities

Verify that each equation is an identity.

1. $\dfrac{\csc x}{\cot x + \tan x} = \cos x$

$$\frac{\csc x}{\cot x + \tan x} = \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\cos x}{\cos^2 x + \sin^2 x} = \frac{\cos x}{1} = \cos x$$

2. $\dfrac{1}{\sin y - 1} - \dfrac{1}{\sin y + 1} = -2 \sec^2 y$

$$\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = \frac{\sin y + 1 - \sin y + 1}{\sin^2 y - 1} = \frac{2}{-\cos^2 y} = -2 \sec^2 y$$

3. $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

$$\sin^3 x - \cos^3 x = (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)$$
$$= (\sin x - \cos x)(1 + \sin x \cos x)$$
$$= (1 + \sin x \cos x)(\sin x - \cos x)$$

4. $\tan \theta + \dfrac{\cos \theta}{1 + \sin \theta} = \sec \theta$

$$\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{\sin \theta + \sin^2 \theta + \cos^2 \theta}{(\cos \theta)(1 + \sin \theta)}$$
$$= \frac{1 + \sin \theta}{(\cos \theta)(1 + \sin \theta)} = \sec \theta$$

Trigonometric Identities

Find a numerical value of one trigonometric function of x.

5. $\sin x \cot x = 1$

$\cos x = 1$

6. $\sin x = 3 \cos x$

$\tan x = 3$

7. $\cos x = \cot x$

$\csc x = 1$ or $\sin x = 1$

8. ***Physics*** The work done in moving an object is given by the formula $W = Fd \cos \theta$, where d is the displacement, F is the force exerted, and θ is the angle between the displacement and the force. Verify that $W = Fd \frac{\cot \theta}{\csc \theta}$ is an equivalent formula.

$$W = Fd \frac{\cot \theta}{\csc \theta} = Fd \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}} = Fd \frac{\cos \theta}{\sin \theta} \cdot \sin \theta = Fd \cos \theta$$