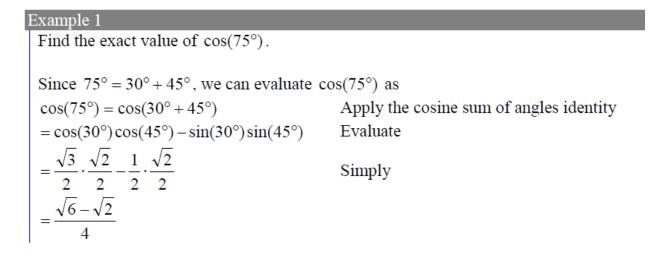
Addition and Subtraction Identities

In this section, we begin expanding our repertoire of trigonometric identities.

| Identities | | |
|--|--|--|
| The sum and difference identities | | |
| $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$ | | |
| $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ | | |
| $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ | | |
| $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$ | | |



Try it Now

2. Find the exact value of $\sin\left(\frac{\pi}{12}\right)$.

| Example 2 | | | |
|-----------|--|--|--|
| | Rewrite $\sin\left(x - \frac{\pi}{4}\right)$ in terms of $\sin(x)$ and $\cos(x)$. | | |
| | $\sin\left(x-\frac{\pi}{4}\right)$ | Use the difference of angles identity for sine | |
| : | $=\sin(x)\cos\left(\frac{\pi}{4}\right) - \cos(x)\sin\left(\frac{\pi}{4}\right)$ | Evaluate the cosine and sine and rearrange | |
| | $=\frac{\sqrt{2}}{2}\sin(x)-\frac{\sqrt{2}}{2}\cos(x)$ | | |

Example 3

Prove $\frac{\sin(a+b)}{\sin(a-b)} = \frac{\tan(a) + \tan(b)}{\tan(a) - \tan(b)}$

As with any identity, we need to first decide which side to begin with. Since the left side involves sum and difference of angles, we might start there

 $\frac{\sin(a+b)}{\sin(a-b)}$ Apply the sum and difference of angle identities $=\frac{\sin(a)\cos(b) + \cos(a)\sin(b)}{\sin(a)\cos(b) - \cos(a)\sin(b)}$

Since it is not immediately obvious how to proceed, we might start on the other side, and see if the path is more apparent.

 $\frac{\tan(a) + \tan(b)}{\tan(a) - \tan(b)}$ Rewriting the tangents using the tangent identity $= \frac{\frac{\sin(a)}{\cos(a)} + \frac{\sin(b)}{\cos(b)}}{\frac{\sin(a)}{\cos(a)} - \frac{\sin(b)}{\cos(b)}}$ Multiplying the top and bottom by $\cos(a)\cos(b)$ $= \frac{\left(\frac{\sin(a)}{\cos(a)} + \frac{\sin(b)}{\cos(b)}\right)\cos(a)\cos(b)}{\left(\frac{\sin(a)}{\cos(a)} - \frac{\sin(b)}{\cos(b)}\right)\cos(a)\cos(b)}$ Distributing and simplifying $= \frac{\sin(a)\cos(a) + \sin(b)\cos(b)}{\sin(a)\cos(a) - \sin(b)\cos(b)}$ From above, we recognize this $= \frac{\sin(a+b)}{\sin(a-b)}$ Establishing the identity

Example 4

Solve
$$\sin(x)\sin(2x) + \cos(x)\cos(2x) = \frac{\sqrt{3}}{2}$$

By recognizing the left side of the equation as the result of the difference of angles identity for cosine, we can simplify the equation

 $\sin(x)\sin(2x) + \cos(x)\cos(2x) = \frac{\sqrt{3}}{2}$ $\cos(x - 2x) = \frac{\sqrt{3}}{2}$ $\cos(-x) = \frac{\sqrt{3}}{2}$ Use the negative angle identity $\cos(x) = \frac{\sqrt{3}}{2}$

Since this is a special cosine value we recognize from the unit circle, we can quickly write the answers:

$$x = \frac{\pi}{6} + 2\pi k$$
, where *k* is an integer
$$x = \frac{11\pi}{6} + 2\pi k$$