Trigonometric Addition and Subtraction Formulas

Trigonometry – Pre-Calculus Addition and Subtraction Identities

1. Find the exact value of $cos 75°$.

2. Find the exact value of $sin\frac{\pi}{12}$.

3. Find the exact value of $sin\frac{7\pi}{12}$.

Trigonometric Addition and Subtraction Formulas

Answers

1. Find the exact value of $cos75°$.

Answer: $\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

Solution:

First, recognize that $75° = 45° + 30°$. Use the rule $\cos(\alpha+\beta) = cos\alpha cos\beta - sin\beta sin\beta$ formula.

$$\cos(45° + 30°) = cos45°cos30° - sin45°sin30°$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

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2. Find the exact value of $sin\frac{\pi}{12}$.

Answer: $\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

Solution:

First, recognize that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$. Use the rule

$$\sin(\alpha - \beta) = sin\alpha cos\beta - cos\alpha sin\beta$$

$$sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = sin\frac{\pi}{3}cos\frac{\pi}{4} - cos\frac{\pi}{3}sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

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3. Find the exact value of $sin\frac{7\pi}{12}$.

Answer: $\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$

Solution:

First, recognize that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$. Use the rule

$$\sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

$$sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = sin\frac{\pi}{3}cos\frac{\pi}{4} + cos\frac{\pi}{3}sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

4. Find the exact value of $sin80°cos20° - cos80°sin20°$.

5. Use the tangent of a sum identity to prove that $\tan(\theta + \pi) = tan\theta$.

6. Find the exact value of $tan15°$.

Answers

4. Find the exact value of $sin80°cos20° - cos80°sin20°$. **Answer:** $\frac{\sqrt{3}}{2}$

Solution:

First, recognize that $sin80°cos20° - cos80°sin20°$ is the formula for $\sin(80° - 20°)$, which is $sin60°$ and $sin60° = \frac{\sqrt{3}}{2}$.

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5. Use the tangent of a sum identity to prove that $\tan(\theta + \pi) = tan\theta$.

Solution:

The identity contains the tangent of a sum, so we will use the formula

$$\tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha tan\beta}$$

$$\tan(\theta + \pi) = \frac{tan\theta + tan\pi}{1 - tan\theta tan\pi}$$

Next, notice that $tan\pi = \frac{sin\pi}{cos\pi} = 0$.

$$\frac{tan\theta + 0}{1 - tan\theta \cdot 0} = \frac{tan\theta}{1} = tan\theta$$

Answer: Proven because the left side equals the right side.

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6. Find the exact value of $tan15°$. **Answer:** $\frac{3-\sqrt{3}}{3+\sqrt{3}}$

Solution:

First, recognize that $15° = 45° - 30°$. The given angle can be written as the tangent of a difference $(\tan(45° - 30°)$, so we will use the formula

$$\tan(\alpha - \beta) = \frac{tan\alpha - tan\beta}{1 + tan\alpha tan\beta}$$

$$\tan(45° - 30°) = \frac{tan45° - tan30°}{1 + tan45°tan30°}$$

$$\frac{tan45° - tan30°}{1 + tan45°tan30°} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

7. Find the exact value of $\sin \frac{17\pi}{12}$.

8. Find the exact value of $cos\frac{\pi}{12}cos\frac{5\pi}{12} + sin\frac{5\pi}{12}sin\frac{\pi}{12}$.

9. Find the exact value of $\frac{tan40° - tan10°}{1 + tan40° tan10°}$.

Answers

7. Find the exact value of $\sin\frac{17\pi}{12}$.

Answer: $-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

First, recognize that $\frac{17\pi}{12}=\frac{2\pi}{3}+\frac{3\pi}{4}$. Use the rule

$$\sin(\alpha+\beta)=sin\alpha cos\beta+cos\alpha sin\beta$$

$$sin\frac{17\pi}{12}=\sin\left(\frac{2\pi}{3}+\frac{3\pi}{4}\right)=sin\frac{2\pi}{3}cos\frac{3\pi}{4}+cos\frac{2\pi}{3}sin\frac{3\pi}{4}$$

$$=\frac{\sqrt{3}}{2}\cdot-\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$=-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$$

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8. Find the exact value of $cos\frac{\pi}{12}cos\frac{5\pi}{12}+sin\frac{5\pi}{12}sin\frac{\pi}{12}$.

Answer: $\frac{1}{2}$

Solution:

First, recognize that $cos\frac{\pi}{12}cos\frac{5\pi}{12}+sin\frac{5\pi}{12}sin\frac{\pi}{12}$ is the formula for $\cos\left(\frac{\pi}{12}-\frac{5\pi}{12}\right)$, which is $cos\left(-\frac{\pi}{3}\right)$ and using the even-angle identity, $cos\left(-\frac{\pi}{3}\right)=cos\left(\frac{\pi}{3}\right)$, which is $\frac{1}{2}$.

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9. Find the exact value of $\frac{tan40°-tan10°}{1+tan40°tan10°}$.

Answer: $\frac{\sqrt{3}}{3}$

Solution:

First, recognize that $\frac{tan40°-tan10°}{1+tan40°tan10°}$ is the formula for $\tan(40°-10°)$ which is $tan30°$. From the unit circle, we see that $tan30°=\frac{y}{x}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

10. Prove that $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2sin\alpha cos\beta$.

Answers

10. Prove that $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2sin\alpha cos\beta$.

Use the sine of a sum and sine of a difference formulas.

$$\sin(\alpha+\beta)=sin\alpha cos\beta+cos\alpha sin\beta$$

$$\sin(\alpha-\beta)=sin\alpha cos\beta-cos\alpha sin\beta$$

Thus,

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=sin\alpha cos\beta+cos\alpha sin\beta+sin\alpha cos\beta-cos\alpha sin\beta$$

Cancel opposite terms.

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=sin\alpha cos\beta+sin\alpha cos\beta$$

$$\sin(\alpha+\beta)+\sin(\alpha-\beta)=2sin\alpha cos\beta$$

Answer: The result is the left side equals the right side.