Trigonometric Equations

Solve each of the following equations.

1.) $\sin x = -\sin^2 x$

Answers

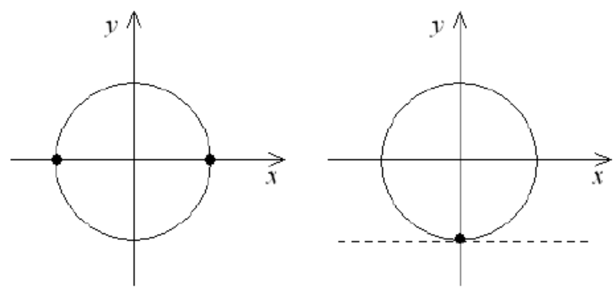
1.) $\sin x = -\sin^2 x$

Solution:

$$\begin{aligned} \sin x &= -\sin^2 x && \text{add } \sin^2 x \\ \sin^2 x + \sin x &= 0 && \text{factor out } \sin x \\ \sin x (\sin x + 1) &= 0 \end{aligned}$$

$$\sin x = 0 \qquad \text{or} \qquad \sin x + 1 = 0$$

$$\sin x = -1$$



If $\sin x = 0$, then $x = k\pi$ where $k \in \mathbb{Z}$. If $\sin x + 1 = 0$, then $\sin x = -1$ and so $x = -\dfrac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

2.) $2\cos^2 x - 5\cos x = 3$

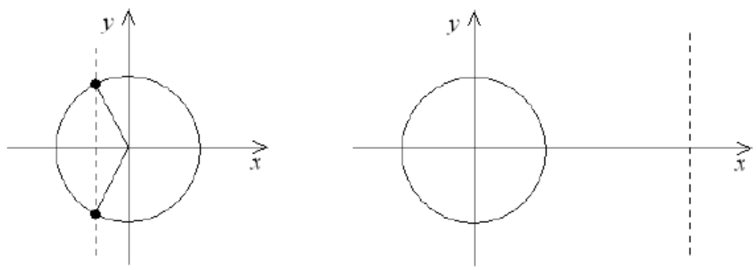
Answers

2.) $2\cos^2 x - 5\cos x = 3$

Solution: This equaton is quadratic in $\cos x$. If helps, we may introduce a new variable, $a = \cos x$.

$$\begin{aligned} 2\cos^2 x - 5\cos x &= 3 \qquad \text{Let } a = \cos x \\ 2a^2 - 5a &= 3 \\ 2a^2 - 5a - 3 &= 0 \\ (2a+1)(a-3) &= 0 \implies a = -\frac{1}{2} \text{ or } a = 3 \end{aligned}$$

$$\cos x = -\frac{1}{2} \qquad \text{or} \qquad \cos x = 3$$



The solution of $\cos x = -\frac{1}{2}$ is $x = \pm\frac{2\pi}{3} + 2k\pi$. There is no solution of $\cos x = 3$.

Solve each of the following equations.

3.) $3(1 - \sin x) = 2\cos^2 x$

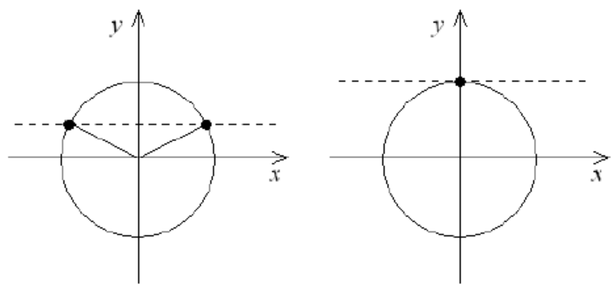
Answers

3.) $3(1-\sin x) = 2\cos^2 x$

Solution: After we write $\cos^2 x = 1 - \sin^2 x$, this equation wil become quadratic in $\sin x$.

$$\begin{array}{rcll} 3(1-\sin x) & = & 2\cos^2 x & \cos^2 x = 1-\sin^2 x \\ 3(1-\sin x) & = & 2\left(1-\sin^2 x\right) & \text{distribute} \\ 3-3\sin x & = & 2-2\sin^2 x & \text{add } 2\sin^2 x \\ 2\sin^2 x - 3\sin x + 3 & = & 2 & \text{subtract } 2 \\ 2\sin^2 x - 3\sin x + 1 & = & 0 & \text{factor} \\ (2\sin x - 1)(\sin x - 1) & = & 0 & \end{array}$$

$$\sin x = \frac{1}{2} \qquad \text{or} \qquad \sin x = 1$$



The solutoin of $\sin x = \dfrac{1}{2}$ is $x = \dfrac{\pi}{6} + 2k\pi$ and $x = \dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$. The solution of $\sin x = 1$ is x where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

4.) $\sin x = -\cos x$

Answers

4.) $\sin x = -\cos x$

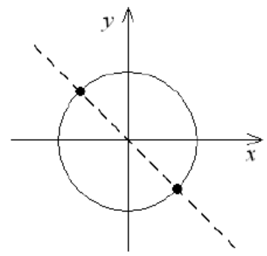
Solution: Since neither $\sin x$ nor $\cos x$ is squared, it is difficult to eliminate either one of them. There are several methods to solve this equation. One method involves squaring both sides of the two equation and then eliminating one trigonometric-function in terms of the other. Notice that, because of the squaring, this method creates extraneous solutions so we have to carefully check our solution. The method presented here is a clever alternative.

Case 1. If $\cos x = 0$

If $\cos x = 0$, then clearly $\sin x = \pm 1$ since $\sin x = \pm\sqrt{1-\cos^2 x}$. Then x is clearly not a solution of the equation $\sin x = -\cos x$.

Case 2. If $\cos x \neq 0$, then we can divide by it.

$$\begin{aligned} \sin x &= -\cos x \qquad \text{divide by } \cos x \neq 0 \\ \frac{\sin x}{\cos x} &= -1 \\ \tan x &= -1 \end{aligned}$$



The solution is $x = -\frac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

5.) $\tan^2 x = \dfrac{1}{3}$

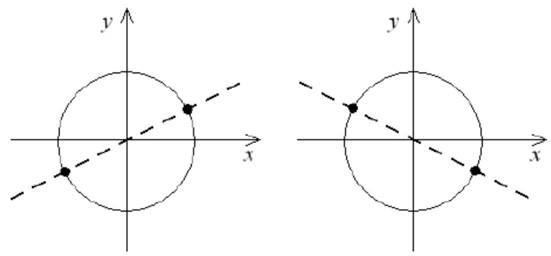
Answers

5.) $\tan^2 x = \frac{1}{3}$

Solution:

$$\begin{aligned} \tan^2 x &= \frac{1}{3} \\ \tan x &= \pm\frac{1}{\sqrt{3}} \end{aligned}$$

$$\tan x = \frac{1}{\sqrt{3}} \quad \text{or} \quad \tan x = -\frac{1}{\sqrt{3}}$$



The solution of $\tan x = \frac{1}{\sqrt{3}}$ is $x = \frac{\pi}{6} + k\pi$ where $k \in \mathbb{Z}$, and the solution of $\tan x = -\frac{1}{\sqrt{3}}$ is $x = -\frac{\pi}{6} + k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

6.) $\cos x \sin x = \cos x$

Answers

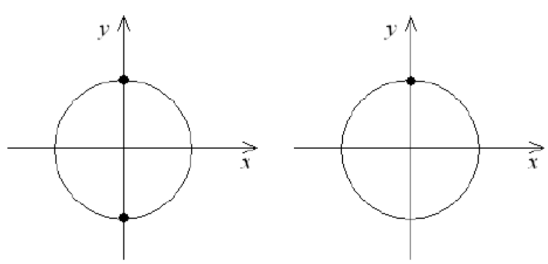
6.) $\cos x \sin x = \cos x$

Solution:

$$\begin{aligned} \cos x \sin x &= \cos x && \text{subtract } \cos x \\ \cos x \sin x - \cos x &= 0 && \text{factor out } \cos x \\ \cos x(\sin x - 1) &= 0 \end{aligned}$$

$$\cos x = 0 \quad \text{or} \quad \sin x - 1 = 0$$

$$\cos x = 0 \quad \text{or} \quad \sin x = 1$$



The solution of $\cos x = 0$ is $x = \pm\frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$ (notice this can be written simpler, as $x = \frac{\pi}{2} + k\pi$) and the solution of $\sin x = \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$. As the picture already indicates, the second case does not bring in any new solutions; if $\sin x = 1$ then of course also $\cos x = 0$. The final answer is $x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

7.) $\tan x = \tan^2 x$

8.) $2\cos x - \sin x + 2\cos x \sin x = 1$

9.) $\cos x = 1 + \sin^2 x$

10.) $7\sin x + 5 = 2\cos^2 x$

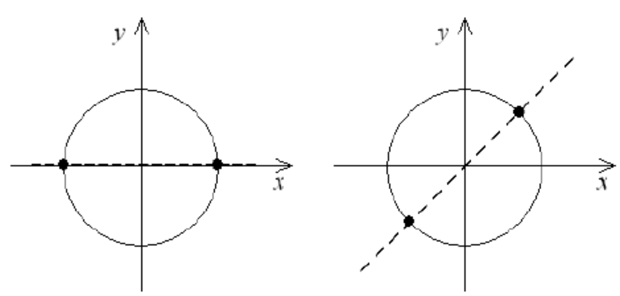
Answers

7.) $\tan x = \tan^2 x$

Solution:

$$\begin{aligned} \tan x &= \tan^2 x && \text{subtract } \tan x \\ 0 &= \tan^2 x - \tan x && \text{factor out } \tan x \\ 0 &= \tan x\,(\tan x - 1) \end{aligned}$$

$$\tan x = 0 \quad \text{or} \quad \tan x = 1$$



The solution of $\tan x = 0$ is $x = k\pi$ where $k \in \mathbb{Z}$, and the solution of $\tan x = 1$ is $x = \dfrac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

8.) $2\cos x - \sin x + 2\cos x \sin x = 1$

Answers

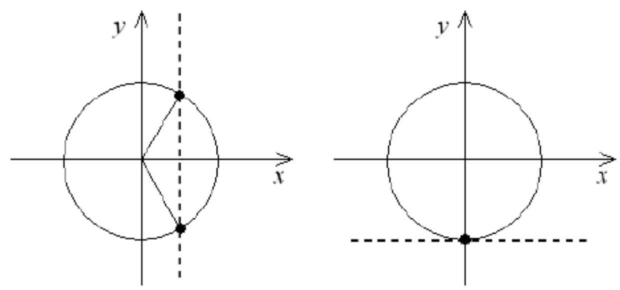
8.) $2\cos x - \sin x + 2\cos x \sin x = 1$

Solution:

$$\begin{aligned} 2\cos x - \sin x + 2\cos x \sin x &= 1 && \text{subtract 1} \\ 2\cos x \sin x + 2\cos x - \sin x - 1 &= 0 && \text{factor by grouping} \\ 2\cos x\,(\sin x + 1) - (\sin x + 1) &= 0 \\ (2\cos x - 1)(\sin x + 1) &= 0 \end{aligned}$$

$$2\cos x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$



If $\cos x = \frac{1}{2}$, then $x = \pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$. And if $\sin x = -1$, then $x = -\frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$.

Trigonometric Equations

Solve each of the following equations.

9.) $\cos x = 1 + \sin^2 x$

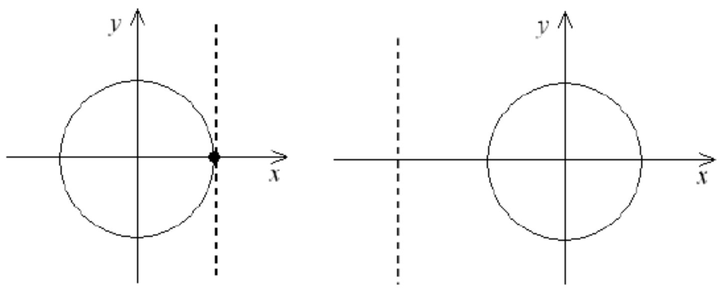
Answers

9.) $\cos x = 1 + \sin^2 x$

Solution:

$$\begin{aligned} \cos x &= 1 + \sin^2 x & \quad & \sin^2 x = 1 - \cos^2 x \\ \cos x &= 1 + 1 - \cos^2 x & & \text{add } \cos^2 x \\ \cos^2 x + \cos x &= 2 & & \text{subtract } 2 \\ \cos^2 x + \cos x - 2 &= 0 & & \text{factor} \\ (\cos x - 1)(\cos x + 2) &= 0 \end{aligned}$$

$\cos x = 1$ or $\cos x = -2$



The solution of $\cos x = 1$ is $x = 2k\pi$ where $k \in \mathbb{Z}$, and the equation $\cos x = -2$ has no solution.

Trigonometric Equations

Solve each of the following equations.

10.) $7\sin x + 5 = 2\cos^2 x$

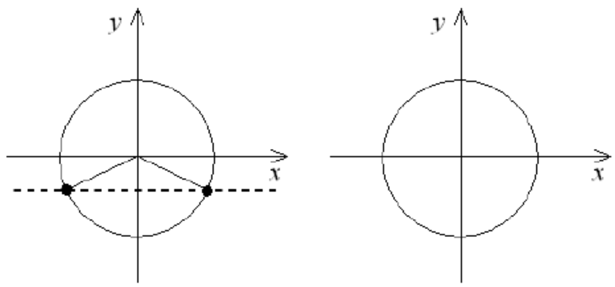
Answers

10.) $7\sin x + 5 = 2\cos^2 x$

Solution:

$$\begin{aligned}
7\sin x + 5 &= 2\cos^2 x && \cos^2 x = 1 - \sin^2 x \\
7\sin x + 5 &= 2\left(1 - \sin^2 x\right) && \text{distribute} \\
7\sin x + 5 &= 2 - 2\sin^2 x && \text{add } 2\sin^2 x \\
2\sin^2 x + 7\sin x + 5 &= 2 && \text{subtract } 2 \\
2\sin^2 x + 7\sin x + 3 &= 0 && \text{factor} \\
(2\sin x + 1)(\sin x + 3) &= 0
\end{aligned}$$

$$\sin x = -\frac{1}{2} \qquad \text{or} \qquad \sin x = -3$$



The solution of $\sin x = -\frac{1}{2}$ is $x = -\frac{\pi}{6} + 2k\pi$ and $x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$, and the equation $\cos x =$ solution.