Exponents and Logarithms … Set 2

Exponential and Logarithmic Equations

1. Solve $log_8 4x = 2 + log_8(2x - 3)$.

2. Solve $log_7(x + 4) = 3 + log_7 x$.

3. Solve: $\ln(10 - 2x^2) = ln2 + ln5$.

Exponents and Logarithms … Set 2

Answers

1. Solve: $log_8 4x = 2 + log_8(2x - 3)$.

Let's solve this equation by exponentiating both sides of the equation with a base of 8 to eliminate the logarithms and using the product rule for exponents.

$$8^{log_8 4x} = 8^{2+log_8(2x-3)} = 8^2 \cdot 8^{log_8(2x-3)}$$

$$4x = 64(2x - 3)$$

$$4x = 128x - 192$$

$$-124x = -192$$

$$x = \frac{192}{124} = \frac{48}{31}$$

Answer: $\frac{48}{31}$

2. Solve: $log_7(x + 4) = 3 + log_7 x$.

Let's solve this equation by combining the logarithms first and then exponentiating both sides of the equation with a base of 7 to eliminate the logarithms.

$$log_7(x + 4) - log_7 x = 3$$

$$log_7\left(\frac{x + 4}{x}\right) = 3$$

$$7^{log_7\left(\frac{x+4}{x}\right)} = 7^3$$

$$\frac{x + 4}{x} = 343; \;\; 343x = x + 4$$

$$342x = 4; \;\; x = \frac{4}{342} = \frac{2}{171}$$

Answer: $\frac{2}{171}$

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3. Solve: $\ln(10 - 2x^2) = ln2 + ln5$.

Let's combine the natural logarithms on the right side of the equation using logarithmic rules and then eliminate the logarithms.

$$\ln(10 - 2x^2) = \ln(2 \cdot 5) = ln10$$

$$e^{\ln(10-2x^2)} = e^{ln10}$$

$$10 - 2x^2 = 10; \;\; -2x^2 = 0; \;\; x = 0$$

Answer: 0

4. Solve: $log3 + logx = 2$.

5. Solve: $log8 + logx = 1$.

6. Solve: $log3x^2 - log3 = 2$.

Exponents and Logarithms … Set 2

Answers

4. Solve: $log3 + logx = 2$.

Let's solve this equation by exponentiating both sides using a base of 10 to eliminate the logarithms.

$$10^{log3+logx} = 10^2$$

$$10^{log3} \cdot 10^{logx} = 100$$

$$3x = 100; \;\; x = \frac{100}{3} \text{ or } 33\frac{1}{3}$$

Answer: $\frac{100}{3}$ or $33\frac{1}{3}$

5. Solve: $log8 + logx = 1$.

Let's combine the logarithms and then exponentiate both sides of the equation with a base of 10 to eliminate the logarithms.

$$log8x = 1$$

$$10^{log8x} = 10^1; \;\; 8x = 10; \;\; x = \frac{10}{8} = \frac{5}{4}$$

Answer: $\frac{5}{4}$

6. Solve: $log3x^2 - log3 = 2$.

Let's combine the logarithms on the left side of the equation and then exponentiate with a base of 10 to eliminate the logarithms.

$$log\left(\frac{3x^2}{3}\right) = 2; \;\; log(x^2) = 2$$

$$10^{log(x^2)} = 10^2; \;\; x^2 = 100; \;\; x = \pm 10$$

Since neither -10 nor 10 cause the logarithm of a negative number, both numbers are accepted.

Answer: ± 10

Exponents and Logarithms … Set 2

7. Solve: $\ln(5-2x) = 4 - ln9$.

8. Solve: $\ln(3x-1) = ln15 - ln4$.

9. Solve: $\ln(4-4x) = ln5 - ln33$.

Exponents and Logarithms … Set 2

Answers

7. Solve: $\ln(5-2x) = 4 - ln9$.

Let's exponentiate both sides with a base of e to eliminate the natural logarithms.
Then, use the product rule for exponents to simplify the equation.

$$e^{\ln(5-2x)} = e^{4-ln9}$$

$$e^{\ln(5-2x)} = e^{4-ln9} = e^4 \cdot e^{ln9^{-1}}$$

$$5 - 2x = e^4 \cdot 9^{-1} = \frac{e^4}{9}$$

Solve for x by subtracting 5 and dividing by -2.

$$-2x = \frac{e^4}{9} - 5 = \frac{e^4 - 45}{9}$$

$$x = \frac{-e^4 + 45}{18}$$

This value does not cause the natural logarithm of a negative number. **Answer**: $\frac{-e^4+45}{18}$

8. Solve: $\ln(3x-1) = ln15 - ln4$.

Let's begin by combining the natural logarithms on the right side of the equation and then eliminating the logarithms.

$$\ln(3x-1) = ln\frac{15}{4};\;\; 3x - 1 = \frac{15}{4}$$

Solve for x by adding 1 and dividing by 3.

$$3x = \frac{15}{4} + 1 = \frac{19}{4};\;\; x = \frac{19}{12}$$

Answer: $\frac{19}{12}$

9. Solve: $\ln(4-4x) = ln5 - ln33$.

Let's begin by combining the natural logarithms on the right side of the equation and then eliminating the logarithms.

$$\ln(4-4x) = ln\frac{5}{33};\;\; 4 - 4x = \frac{5}{33}$$

Solve for x by subtracting 4 and dividing by -4.

$$-4x = \frac{5}{33} - 4 = \frac{5}{33} - \frac{1324}{33} - \frac{127}{33}$$

$$x = -\frac{127}{33} \cdot -\frac{1}{4} = \frac{127}{132}$$

Answer: $\frac{127}{132}$

Exponents and Logarithms … Set 2

10. Solve: $log_3(4x - 2) = log_3(5 - 5x)$.

11. Solve: $log_7(4x - 5) = log_7(2x - 1)$.

12. Solve: $10 - log_3(x + 3) = 10$.

Exponents and Logarithms … Set 2

Answers

10. Solve: $log_3(4x-2) = log_3(5-5x)$.

Since both sides of the equation contain a logarithm with the same base, exponentiate with a base of 3 to eliminate the logarithms.

$$3^{log_3(4x-2)} = 3^{log_3(5-5x)}$$

$$4x - 2 = 5 - 5x$$

Solve for x, which gives $x = \frac{7}{9}$

Answer: $\frac{7}{9}$

11. Solve: $log_7(4x-5) = log_7(2x-1)$.

Since both sides of the equation contain a logarithm with the same base, exponentiate with a base of 7 to eliminate the logarithms.

$$7^{log_7(4x-5)} = 7^{log_7(2x-1)}$$

$$4x - 5 = 2x - 1$$

Solve for x, which gives $x = 2$.

Answer: 2

12. Solve: $10 - log_3(x+3) = 10$.

Let's begin by subtracting 10 from both sides and divide both sides by 0.

$$-log_3(x+3) = 10 - 10 = 0;\ \ log_3(x+3) = 0$$

Now, exponentiate both sides with a base of 3 to eliminate the logarithm and solve for x.

$$3^{log_3(x+3)} = 3^0;\ \ x + 3 = 1;\ \ x = -2$$

The negative answer does not cause a logarithm of a negative number.

Answer: -2

Exponents and Logarithms … Set 2

13. Solve: $\ln(x^2 + 12) = \ln(-9x - 2)$.

14. Solve: $log_3(x + 6) = log_3 2 + log_3 x$.

15. Solve: $log_5(x + 1) = log_5 29 + log_5 x$.

Exponents and Logarithms … Set 2

Answers

13. Solve: $\ln(x^2 + 12) = \ln(-9x - 2)$.

Since both sides of the equation are the natural logarithm, exponentiate with a base of e to eliminate the natural logarithm.

$$e^{\ln(x^2+12)} = e^{\ln(-9x-2)}$$

$$x^2 + 12 = -9x - 2$$

$$x^2 + 9x + 14 = 0$$

$$(x + 2)(x + 7) = 0$$

Solve for x by factoring, and $x = -2, x = -7$. Neither answer is extraneous.

Answer: $-7, -2$

14. Solve: $log_3(x + 6) = log_3 2 + log_3 x$.

Let's solve this equation by combining the logarithms on the right side of the equation and then exponentiating with a base of 3 to eliminate the logarithms. Then, solve for x.

$$log_3(x + 6) = log_3 2x$$

$$3^{log_3(x+6)} = 3^{log_3 2x}$$

$$x + 6 = 2x; \;\; x = 6$$

Answer: 6

15. Solve: $log_5(x + 1) = log_5 29 + log_5 x$.

Let's combine the logarithms on the right side of the equation and then exponentiate with a base of 5 to eliminate the logarithms. Then, solve for x.

$$log_5(x + 1) = log_5 29x$$

$$5^{log_5(x+1)} = 5^{log_5 29x}$$

$$x + 1 = 29x; \;\; 1 = 28x; \;\; x = \frac{1}{28}$$

Answer: $\frac{1}{28}$

Exponents and Logarithms … Set 2

16. Solve: $log_9 6 + log_9 2x^2 = log_9 48$.

17. Solve: $\ln(x - 3) = \ln(x - 5) + ln5$.

18. Solve: $2logx = log37 - log7$.

Exponents and Logarithms … Set 2

Answers

16. Solve: $log_9 6 + log_9 2x^2 = log_9 48$.

Let's combine the logarithms on the left side. Then, exponentiate both sides with a base of 9 to eliminate the logarithms. Next, solve for x.

$$log_9 12x^2 = log_9 48$$

$$9^{log_9 12x^2} = 9^{log_9 48}$$

$$12x^2 = 48;\ \ 12x^2 - 48 = 0;\ \ 12(x^2 - 4) = 0$$

$$12(x + 2)(2 - 2) = 0;\ \ x = -2, x = 2$$

Neither number causes us to take the logarithm of a negative number.

Answer: $-2, 2$

17. Solve: $\ln(x - 3) = \ln(x - 5) + ln5$.

Let's combine the natural logarithms on the right side of the equation. Then, exponentiate both sides of the equation with a base of e.

$$\ln(x - 3) = \ln[5(x - 5)]$$

$$e^{\ln(x-3)} = e^{\ln[5(x-5)]}$$

$$x - 3 = 5(x - 5)$$

$$x - 3 = 5x - 25;\ \ 4x = 22; x = \frac{22}{4} = \frac{11}{2}$$

Solving for x gives $x = \frac{11}{2}$

Answer: $\frac{11}{2}$

Answers

18. Solve: $2logx = log37 - log7$.

Let's combine the logarithms on the right side of the equation and apply the exponent rule for logarithms on the left side.

$$logx^2 = log\frac{37}{7}$$

Exponentiate both sides with a base of 10.

$$10^{logx^2} = 10^{log\frac{37}{7}};\ \ x^2 = \frac{37}{7}$$

Solving for x gives $x = -\sqrt{\frac{37}{7}}$, $x = \sqrt{\frac{37}{7}}$. The negative value causes the logarithm of a negative number, so that answer is eliminated.

Answer: $\sqrt{\frac{37}{7}}$

Answers

19. Solve: $\log(2x + 1) = 1 + \log(x - 2)$.

20. Solve: $logx + \log(x + 15) = 2$.

19. Solve: $\log(2x+1) = 1 + \log(x-2)$.

Let's exponentiate both sides of the equation with a base of 10 and use the product rule for exponents to simplify the equation.

$$10^{\log(2x+1)} = 10^{1+\log(x-2)}$$

$$10^{\log(2x+1)} = 10^1 \cdot 10^{\log(x-2)}$$

$$2x + 1 = 10(x-2)$$

$$2x + 1 = 10x - 20$$

Solve for x, which gives $x = \frac{21}{8}$.

Answer: $\frac{21}{8}$

20. Solve: $logx + \log(x+15) = 2$.

Let's solve this equation by combining the logarithms on the left side of the equation and then exponentiating the equation with a base of 10 to eliminate the logarithm.

$$\log[x(x+15)] = 2$$

$$10^{\log[x(x+15)]} = 10^2$$

$$x(x+15) = 100; \;\; x^2 + 15x = 100$$

$$x^2 + 15x - 100 = 0$$

$$(x+20)(x-5) = 0$$

Solve for x and we find that $x = -20, x = 5$.

The value -20 causes us to take the logarithm of a negative number, so that answer is eliminated.

Answer: 5