Exponents and Logarithms … Set 3

Worksheet: Logarithmic Function

1. Find the value of y.

(1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \frac{1}{8} = y$

(5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \frac{1}{7} = y$ (8) $\log_3 \frac{1}{9} = y$

(9) $\log_y 32 = 5$ (10) $\log_9 y = -\frac{1}{2}$ (11) $\log_4 \frac{1}{8} = y$ (12) $\log_9 \frac{1}{81} = y$

2. Evaluate.

(1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (3) $\log_{25} 5^3$ (4) $16^{\log_4 8}$

3. Write the following expressions in terms of logs of x, y and z.

(1) $\log x^2 y$ (2) $\log \frac{x^3 y^2}{z}$ (3) $\log \frac{\sqrt{x}\sqrt[3]{y^2}}{z^4}$ (4) $\log xyz$

(5) $\log \frac{x}{yz}$ (6) $\log \left(\frac{x}{y}\right)^2$ (7) $\log (xy)^{\frac{1}{3}}$ (8) $\log x\sqrt{z}$

(9) $\log \frac{\sqrt[3]{x}}{\sqrt[3]{yz}}$ (10) $\log \sqrt[4]{\frac{x^3 y^2}{z^4}}$ (11) $\log x\sqrt{\frac{\sqrt{x}}{z}}$ (12) $\log \sqrt{\frac{xy^2}{z^8}}$

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Answers

1. (1) 2

   (2) 0

   (3) $\frac{1}{2}$

   (4) -3

   (5) 0

   (6) 3

   (7) -1

   (8) -2

   (9) 2

   (10) $\frac{1}{3}$

   (11) $-\frac{3}{2}$

   (12) -2

2. (1) 0

   (2) 1

   (3) 3

   (4) 3

   (5) $\frac{3}{2}$

   (6) 64

3. (1) $2\log x + \log y$

   (2) $3\log x + 2\log y - \log z$

   (3) $\frac{1}{2}\log x + \frac{2}{3}\log y - 4\log z$

   (4) $\log x + \log y + \log z$

   (5) $\log x - \log y - \log z$

   (6) $2\log x - 2\log y$

   (7) $\frac{1}{3}\log x + \frac{1}{3}\log y$

   (8) $\log x + \frac{1}{2}\log z$

   (9) $\frac{1}{3}(\log x - \log y - \log z)$

   (10) $\frac{1}{4}\log x + \frac{1}{2}\log y - \log z$

   (11) $\frac{5}{4}\log x - \frac{1}{2}\log z$

   (12) $\frac{1}{2}\log x + \log y - 4\log z$

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4. Write the following equalities in exponential form.

(1) $\log_3 81 = 4$ (2) $\log_7 7 = 1$ (3) $\log_{\frac{1}{2}} \frac{1}{8} = 3$ (4) $\log_3 1 = 0$

(5) $\log_4 \frac{1}{64} = -3$ (6) $\log_6 \frac{1}{36} = -2$ (7) $\log_x y = z$ (8) $\log_m n = \frac{1}{2}$

5. Write the following equalities in logarithmic form.

(1) $8^2 = 64$ (2) $10^3 = 10000$ (3) $4^{-2} = \frac{1}{16}$ (4) $3^{-4} = \frac{1}{81}$

(5) $\left(\frac{1}{2}\right)^{-5} = 32$ (6) $\left(\frac{1}{3}\right)^{-3} = 27$ (7) $x^{2z} = y$ (8) $\sqrt{x} = y$

6. True or False?

(1) $\log\left(\frac{x}{y^3}\right) = \log x - 3\log y$ (2) $\log(a-b) = \log a - \log b$ (3) $\log x^k = k \cdot$

(4) $(\log a)(\log b) = \log(a+b)$ (5) $\frac{\log a}{\log b} = \log(a-b)$ (6) $(\ln a)^k = k$

(7) $\log_a a^a = a$ (8) $-\ln\left(\frac{1}{x}\right) = \ln x$ (9) $\ln_{\sqrt{x}} x^k = 2$

7. Solve the following logarithmic equations.

(1) $\ln x = -3$ (2) $\log(3x-2) = 2$

(3) $2\log x = \log 2 + \log(3x-4)$ (4) $\log x + \log(x-1) = \log(4x)$

(5) $\log_3(x+25) - \log_3(x-1) = 3$ (6) $\log_9(x-5) + \log_9(x+3) = 1$

(7) $\log x + \log(x-3) = 1$ (8) $\log_2(x-2) + \log_2(x+1) = 2$

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Answers

4. (1) $3^4 = 81$

   (2) $7^1 = 7$

   (3) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

   (4) $3^0 = 1$

   (5) $4^{-3} = \frac{1}{64}$

   (6) $6^{-2} = \frac{1}{36}$

   (7) $x^z = y$

   (8) $m^{\frac{1}{2}} = n$

5. (1) $\log_8 64 = 2$

   (2) $\log_{10} 10000 = 3$

   (3) $\log_4 \frac{1}{16} = -2$

   (4) $\log_3 \frac{1}{81} = -4$

   (5) $\log_{\frac{1}{2}} 32 = -5$

   (6) $\log_{\frac{1}{3}} 27 = -3$

   (7) $\log_x y = 2z$

   (8) $\log_x y = \frac{1}{2}$

6. (1) True

   (2) False

   (3) True

   (4) False

   (5) False

   (6) False

   (7) True

   (8) True

7. (1) $S = \{e^{-3}\}$

   (2) $S = \{34\}$

   (3) $S = \{2, 4\}$

   (4) $S = \{5\}$

   (5) $S = \{2\}$

   (6) $S = \{6\}$

   (7) $S = \{5\}$

   (8) $S = \{3\}$

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8. Prove the following statements.

   (1) $\log_{\sqrt{b}} x = 2\log_b x$ (2) $\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$ (3) $\log_{b^4} x^2 = \log_b \sqrt{x}$

9. Given that $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, express the following expressions in terms of x, y, and z.

   (1) $\log 12$ (2) $\log 200$ (3) $\log \frac{14}{3}$ (4) $\log 0.3$

   (5) $\log 1.5$ (6) $\log 10.5$ (7) $\log 15$ (8) $\log \frac{6000}{7}$

10. Solve the following equations.

    (1) $3^x - 2 = 12$ (2) $3^{1-x} = 2$

    (3) $4^x = 5^{x+1}$ (4) $6^{1-x} = 10^x$

    (5) $3^{2x+1} = 2^{x-2}$ (6) $\frac{10}{1+e^{-x}} = 2$

    (7) $5^{2x} - 5^x - 12 = 0$ (8) $e^{2x} - 2e^x = 15$

11. Draw the graph of each of the following logarithmic functions, and analyze each of them completely.

    (1) $f(x) = \log x$ (2) $f(x) = \log -x$

    (3) $f(x) = -\log(x-3)$ (4) $f(x) = -2\log_3(3-x)$

    (5) $f(x) = -\ln(x+1)$ (6) $f(x) = 2\ln\frac{1}{2}(x+3)$

    (7) $f(x) = \ln(2x+4)$ (8) $f(x) = -2\ln(-3x+6)$

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Answers

8. (1)

$$\log_{\sqrt{b}} x = 2\log_b x$$

$$\begin{aligned}\log_{\sqrt{b}} x &= \frac{\log x}{\log \sqrt{b}} \\ &= \frac{\log x}{\frac{1}{2}\log b} \\ &= 2\frac{\log x}{\log b} \\ &= 2\log_b x \quad \square\end{aligned}$$

(2)

$$\log_{\frac{1}{\sqrt{b}}} \sqrt{x} = -\log_b x$$

$$\begin{aligned}\log_{\frac{1}{\sqrt{b}}} \sqrt{x} &= \frac{\log \sqrt{x}}{\log \frac{1}{\sqrt{b}}} \\ &= \frac{\frac{1}{2}\log x}{-\frac{1}{2}\log b} \\ &= -\frac{\log x}{\log b} \\ &= -\log_b x \quad \square\end{aligned}$$

(3)

$$\log_{b^4} x^2 = \log_b \sqrt{x}$$

$$\begin{aligned}\log_{b^4} x^2 &= \frac{\log x^2}{\log b^4} \\ &= \frac{2\log x}{4\log b} \\ &= \frac{1}{2}\frac{\log x}{\log b} \\ &= \frac{1}{2}\log_b x \\ &= \log_b \sqrt{x} \quad \square\end{aligned}$$

9. (1) $2x + y$

(2) $x + 2$

(3) $x - y + z$

(4) $y - 1$

(5) $y - x$

(6) $y + z - x$

(7) $1 - x + y$

(8) $x + y - z + 3$

10. (1) $S = \{2.402\}$

(2) $S = \{0.369\}$

(3) $S = \{-7.213\}$

(4) $S = \{0.438\}$

(5) $S = \{-1.652\}$

(6) $S = \{-\ln 4\}$

(7) $S = \{\log_5 4\}$

(8) $S = \{\ln 5\}$

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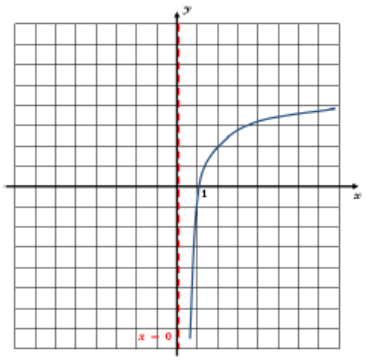
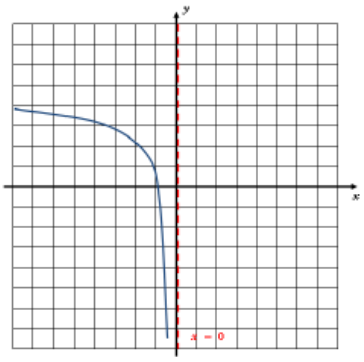
Answers

11. (1)



$\mathrm{Dom}(f) =]0, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in]0, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]0, 1]$
$f(x) \leq 0$ if $x \in [1, +\infty[$

(2)

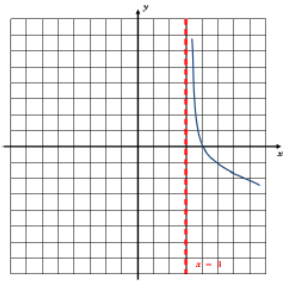


$\mathrm{Dom}(f) =]-\infty, 0[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-\infty, 0[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-\infty, -1]$
$f(x) \leq 0$ if $x \in [-1, 0[$

Exponents and Logarithms … Set 3

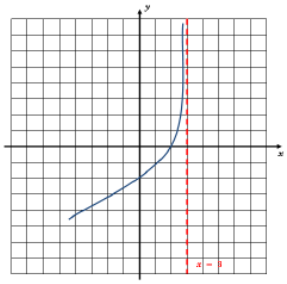
Answers

(3)



$\text{Dom}(f) =]3, +\infty[$
$\text{R}(f) = \mathbb{R}$
Zeros: 4
Y-intercept: None
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]3, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]3, 4]$
$f(x) \leq 0$ if $x \in [4, +\infty[$

(4)

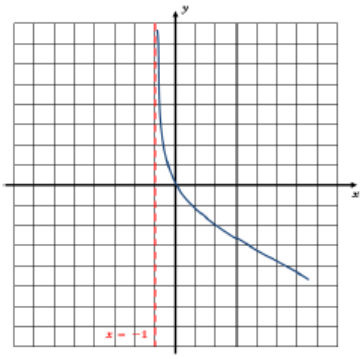


$\text{Dom}(f) =]-\infty, 3[$
$\text{R}(f) = \mathbb{R}$
Zeros: 2
Y-intercept: -2
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 3[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]2, 3[$
$f(x) \leq 0$ if $x \in]-\infty, 2[$

Exponents and Logarithms … Set 3

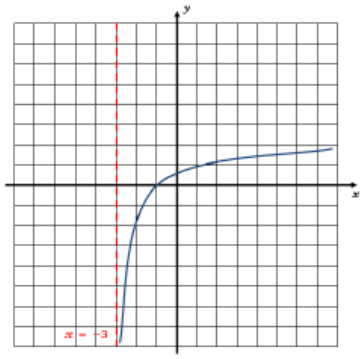
Answers

(5)



$\mathrm{Dom}(f) =]-1, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: 0
Y-intercept: 0
Variation:
$f(x) \nearrow$ if $x \in \emptyset$
$f(x) \searrow$ if $x \in]-1, +\infty[$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in]-1, 0[$
$f(x) \leq 0$ if $x \in]0, +\infty[$

(6)

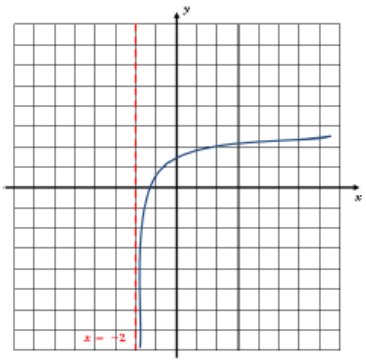


$\mathrm{Dom}(f) =]-3, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1
Y-intercept: $2 \ln \frac{3}{2}$
Variation:
$f(x) \nearrow$ if $x \in]-3, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1, +\infty[$
$f(x) \leq 0$ if $x \in]-3, -1]$

Exponents and Logarithms … Set 3

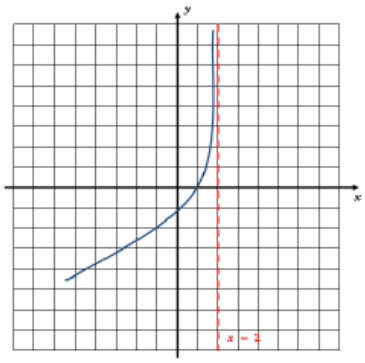
Answers

(7)



$\mathrm{Dom}(f) =]-2, +\infty[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: -1.5
Y-intercept: $\ln 4$
Variation:
$f(x) \nearrow$ if $x \in]-2, +\infty[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [-1.5, +\infty[$
$f(x) \leq 0$ if $x \in]-2, -1.5]$

(8)



$\mathrm{Dom}(f) =]-\infty, 2[$
$\mathrm{R}(f) = \mathbb{R}$
Zeros: $\dfrac{5}{3}$
Y-intercept: $-2\ln 6$
Variation:
$f(x) \nearrow$ if $x \in]-\infty, 2[$
$f(x) \searrow$ if $x \in \emptyset$
Extremums: Max: None, Min: None
Sign:
$f(x) \geq 0$ if $x \in [\frac{5}{3}, 2[$
$f(x) \leq 0$ if $x \in]-\infty, \frac{5}{3}[$

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Answers

12. (1) $f^{-1}(x) = 2^{x+5} + 3$

    (2) $f^{-1}(x) = 3^{\frac{x-1}{3}} - 3$

    (3) $f^{-1}(x) = \frac{1}{2}10^{\frac{2-x}{2}} + 1$

    (4) $f^{-1}(x) = -\frac{1}{2}e^{1-x} + \frac{1}{2}$

    (5) $f^{-1}(x) = \log_2(x+3)$

    (6) $f^{-1}(x) = \frac{1}{3}\log_3\left(\frac{x+1}{2}\right)$

    (7) $f^{-1}(x) = -\ln\left(\frac{2-x}{5}\right)$

    (8) $f^{-1}(x) = -\frac{1}{2}\ln\left(\frac{1-x}{2}\right)$

13. 37 years.

14. 9 years.

15. 53 years old.

16. (a) $f(t) = 10000 \cdot 2^{1.5t}$. Where t is the number of hours.

    (b) 28 284 bacteria.

    (c) 92.88 minutes.

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12. Find the inverse of each of the following functions.

    (1) $f(x) = \log_2(x-3) - 5$

    (2) $f(x) = 3\log_3(x+3) + 1$

    (3) $f(x) = -2\log 2(x-1) + 2$

    (4) $f(x) = -\ln(1-2x) + 1$

    (5) $f(x) = 2^x - 3$

    (6) $f(x) = 2 \cdot 3^{3x} - 1$

    (7) $f(x) = -5 \cdot e^{-x} + 2$

    (8) $f(x) = 1 - 2e^{-2x}$

13. 15 000$ is invested in an account that yeilds 5% interest per year. After how many years will the account be worth 91 221.04$ if the interest is compounded yearly?

14. 8 000$ is invested in an account that yeilds 6% interest per year. After how many years will the account be worth 13709.60$ if the interest is compounded monthly?

15. Starting at the age of 40, an average man loses 5% of his hair every year. At what age should an average man expect to have half his hair left?

16. A bacteria culture starts with 10 00 bacteria and the number doubles every 40 minutes.

    (a) Find a formula for the number of bacteria at time t.

    (b) Find the number of bacteria after one hour.

    (c) After how many minutes will there be 50 000 bacteria?