# Exponents and Logarithms … Set 5

Honors Precalculus Applications of Logarithms & Exponential Functions

YES Calculator is permitted. Show your work and box your answers when appropriate. ROUNDING RULES: Round money to the nearest penny. Round Bacteria to nearest whole. Round any rate to the at least three decimal places. Do not round in the middle of a problem, only at the end of the problem. Provide exact values when requested.

| | |
|---|---|
| 1. Find the amount that results when \$1500 is invested at 8% compounded monthly after a period of 14 years. | 2. Find the amount that results when \$375 is invested at 4% compounded continuously after a period of 3 years. |
| 3. Find the principal needed now (present value) to get \$13,000 after 5 years at 9% compounded quarterly. | 4. Find the effective interest rate of interest for 8.5% compounded continuously. |
| 5. How long does it take for an investment to double in value if it is invested at 3% compounded monthly? | 6. How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of 7% interest compounded continuously. |
| 7. A skillet is removed from an oven whose temperature is $450°$ and placed in a room whose temperature is $70°$. After 5 minutes, the temperature of the skillet is $400°$.<br>a. Write a formula to model the temperature of the skillet after t seconds. Use an exact value for k.<br>b. How long will it be until the skillet is $150°$? | 8. A culture of bacteria obeys the laws of uninhibited growth.<br>a. Write a formula if 600 bacteria are present initially, and there are 790 after 1 hour.<br>b. How many bacteria will be present after 12 hours? |

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## Answers

YES Calculator is permitted. Show your work and box your answers when appropriate. ROUNDING RULES: Round money to the nearest penny. Round Bacteria to nearest whole. Round any rate to the at least three decimal places. Do not round in the middle of a problem, only at the end of the problem. Provide exact values when requested.

1. Find the amount that results when \$1500 is invested at 8% compounded monthly after a period of 14 years.

$1500\left(1+\frac{.08}{12}\right)^{168}$

$\boxed{\$4580.23}$

2. Find the amount that results when \$375 is invested at 4% compounded continuously after a period of 3 years.

$375\,e^{(.04*3)}$

$\boxed{\$422.81}$

3. Find the principal needed now (present value) to get \$13,000 after 5 years at 9% compounded quarterly.

$13{,}000 = P\left(1+\frac{.09}{4}\right)^{20}$

$\boxed{\$8330.61}$

4. Find the effective interest rate of interest for 8.5% compounded continuously.

$r = 1-e^{r}$

$= 1-e^{.085}$

$.08871067$

$\boxed{8.871\%}$

5. How long does it take for an investment to double in value if it is invested at 3% compounded monthly?

$2 = \left(1+\frac{.03}{12}\right)^{12t}$

$\ln 2 = 12t\ln\left(1+\frac{.03}{12}\right)$

$t \approx 23.133775$

$\boxed{\text{About } 23.134 \text{ years}}$

6. How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of 7% interest compounded continuously.

$80{,}000 = 25000\,e^{.07t}$

$\boxed{\text{About } 16.616 \text{ years}}$

7. A skillet is removed from an oven whose temperature is 450° and placed in a room whose temperature is 70°. After 5 minutes, the temperature of the skillet is 400°.

$T(t) = T_s + (T_0 - T_s)e^{-kt}$

a. Write a formula to model the temperature of the skillet after t seconds. Use an exact value for k.

$T(t) = 70 + (450-70)e^{-kt}$

$T(t) = 70 + 380e^{-kt}$ use (5, 400)

$400 = 70 + 380e^{-5k}$

$\frac{\ln\left(\frac{33}{38}\right)}{-5} = k$

$\boxed{T(t) = 70 + 380e^{+0.2\ln\left(\frac{33}{38}\right)t}}$

b. How long will it be until the skillet is 150°?

$150 = 70 + 380e^{+.2\ln\left(\frac{33}{38}\right)t}$

$\ln\left(\frac{4}{19}\right) = +.2\ln\left(\frac{33}{38}\right)t$

$55.22257$

$\boxed{\text{About } 55.223 \text{ minutes}}$

8. A culture of bacteria obeys the laws of uninhibited growth.

a. Write a formula if 600 bacteria are present initially, and there are 790 after 1 hour.

$A = A_0e^{rt}$

$790 = 600e^{r}$

$\frac{790}{600} = e^{r}$

$r = \ln\left(\frac{79}{60}\right)$

$\boxed{A = 600e^{\ln\left(\frac{79}{60}\right)t}}$

b. How many bacteria will be present after 12 hours?

$16287.75925$

$\boxed{\text{About } 16288 \text{ bacteria}}$

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9. A colony of bacteria grows according to the law of uninhibited growth where $N(t)=100e^{0.054t}$ Where N is measured in grams and t is measured in days.
   a. Determine the initial amount of bacteria
   b. What is the growth rate of the bacteria?
   c. What is the population after 5 days?
   d. How long will it take for the population to reach 140 grams?

10. A piece of charcoal is found to contain 25% of the carbon 14 that it originally had. When did the tree from which the charcoal came from die? Remember the half-life of Carbon 14 is about 5600 years. (Do not round any value until the end!)

11. The normal healing of wounds can be modeled by an exponential function. If $A_0$ represents the original area of the wound and if $A$ equals the area of the wound, then the function $A(n)=A_0e^{-0.35n}$ describes the area of a wound after n days following an injury when no infection is present to slow healing. Suppose a wound is initially 80 square millimeters.
   a. After how many days will the wound be half its original size?
   b. How long before the wound is 10% of its original size.

12. Solve. You do not have to show work if calculator is your method.
   $\log_2 x+\log_4 x=\log_3(2-x)$

13. Solve. Round answer to three decimal places
   $e^x-\ln 5=2-x^2$

14. State the domain $y=\log_5(x^2-2x-15)$

15. $f(x)=\log_7(7-x)$
   a. Solve $f(x)=0$
   b. Evaluate $f(0)$

# Exponents and Logarithms … Set 5

Answers

| | |
|---|---|
| 9. A colony of bacteria grows according to the law of uninhibited growth where $N(t)=100e^{0.054t}$ Where N is measured in grams and t is measured in days.<br>a. Determine the initial amount of bacteria<br>$\boxed{100 \text{ Bacteria}}$<br>b. What is the growth rate of the bacteria?<br>$\boxed{5.4\%}$<br>c. What is the population after 5 days?<br>130.9964 $\boxed{\text{About } 131 \text{ Bacteria}}$<br>d. How long will it take for the population to reach 140 grams?<br>$\frac{140}{100}=e^{0.054t}$ $t\approx 6.230967$<br>$\boxed{\text{About 6 to 7 days}}$ 6.231 days. | 10. A piece of charcoal is found to contain 25% of the carbon 14 that it originally had. When did the tree from which the charcoal came from die? Remember the half-life of Carbon 14 is about 5600 years. (Do not round any value until the end!)<br>use.<br>$.25=e^{\frac{\ln(1/2)}{5600}}$<br>$\ln(.25)=\frac{\ln(1/2)}{5600}$<br>$\boxed{\text{About } 11{,}200 \text{ years}}$<br>$\frac{1}{2}=e^{K5600}$<br>$\ln(\frac{1}{2})=5600K$<br>$K\approx\frac{\ln(1/2)}{5600}$<br>$K\approx -0.000123776$ |

11. The normal healing of wounds can be modeled by an exponential function. If $A_0$ represents the original area of the wound and if $A$ equals the area of the wound, then the function $A(n)=A_0e^{-0.35n}$ describes the area of a wound after n days following an injury when no infection is present to slow healing. Suppose a wound is initially 80 square millimeters.

a. After how many days will the wound be half its original size?

$0.5=e^{-0.35n}$

$n\approx 1.98042$

About 1.980 or 2 days.

b. How long before the wound is 10% of its original size.

$0.10=e^{-0.35n}$

$\ln(.1)=-0.35n$

$n\approx 6.5788$

$\boxed{\text{About 6.579 days or 7 days}}$

| | |
|---|---|
| 12. Solve. You do not have to show work if calculator is your method.<br>$\log_2 x+\log_4 x=\log_3(2-x)$<br>In calculator<br>$Y_1=\frac{\ln x}{\ln 2}+\frac{\ln x}{\ln 4}$<br>$Y_2=\frac{\ln(2-x)}{\ln 3}$<br>$\boxed{X=1}$ | 13. Solve. Round answer to three decimal places<br>$e^x-\ln 5=2-x^2$ use calculator<br>$Y_1=e^x-\ln 5$<br>$Y_2=2-x^2$<br>$x\approx -1.858$ or $x\approx 0.977$ |
| 14. State the domain $y=\log_5(x^2-2x-15)$<br>$x^2-2x-15>0$<br>$(x-5)(x+3)>0$<br>-3, 5<br>+ − +<br>$\boxed{(-\infty,-3)\cup(5,\infty)}$ | 15. $f(x)=\log_7(7-x)$<br>a. Solve $f(x)=0$<br>$0=\log_7(7-x)$<br>$7^0=7-x$<br>$1=7-x$<br>$\boxed{x=6}$<br>b. Evaluate $f(0)$<br>$\log_7(7-0)$<br>$\log_7 7=\boxed{1}$ |