

Trigonometry

TRIGONOMETRY

Solve for $0 \leq \theta \leq 2\pi$ without using a calculator:

1. $\sin \theta = -\frac{1}{2}$ 2. $\csc \theta = \sqrt{2}$ 3. $\tan \theta = -1$ 4. $\cos \theta = -2$

Solve for θ in degrees giving all solutions.

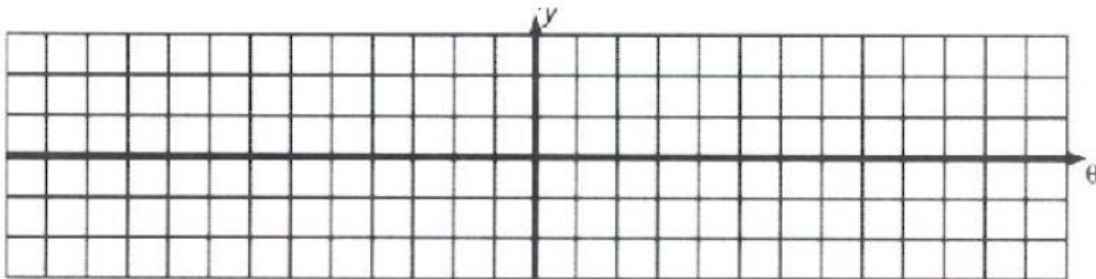
5. $\sin \theta = -1$ 6. $\cos \theta = -\frac{\sqrt{3}}{2}$ 7. $\tan \theta = \text{undefined}$

Give the exact value of each expression.

8. $\tan^{-1} \sqrt{3}$ 9. $\cot \left(\sin^{-1} \frac{1}{2} \right)$ 10. $\sec \left(\cos^{-1} \left(-\frac{3}{5} \right) \right)$

11. Given $y = \frac{1}{3} \sin \frac{1}{2} x$, find

- amplitude
- period
- graph at least two periods of the function



Trigonometry

Answers

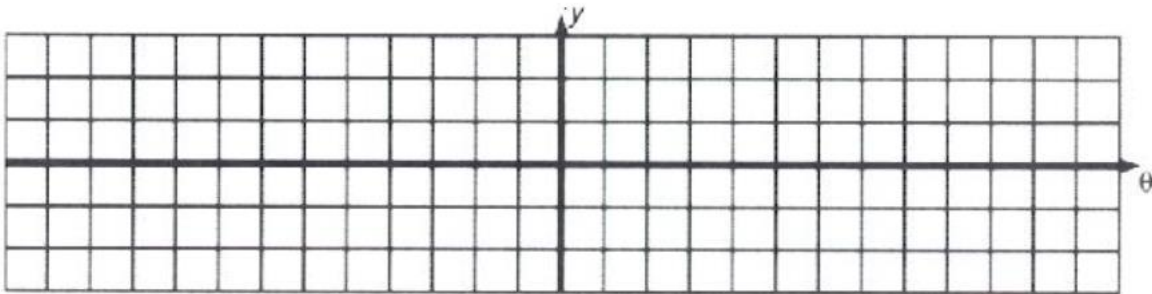
- | | | | | | | | |
|----|-----------------------------------|-----|--|------|----------------------------------|------|---------------------------|
| 1. | $\frac{7\pi}{6}, \frac{11\pi}{6}$ | 2. | $\frac{\pi}{4}, \frac{3\pi}{4}$ | 3. | $\frac{3\pi}{4}, \frac{7\pi}{4}$ | 4. | undefined |
| 5. | $270^\circ \pm 360^\circ n$ | 6. | $150^\circ \pm 360^\circ n$
$210^\circ \pm 360^\circ n$ | 7. | $90^\circ \pm 180^\circ n$ | 8. | $\frac{\pi}{3}, 60^\circ$ |
| 9. | $\sqrt{3}$ | 10. | $-\frac{5}{3}$ | 11a. | $\frac{1}{3}$ | 11b. | 4π |

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12. Given: $y = -4 + 2 \cos 3\left(x - \frac{\pi}{2}\right)$, find

- Amplitude
- Period
- Phase shift
- Vertical shift
- graph at least two periods of the function



13. Simplify: $(1 - \sin \theta)(1 + \csc \theta) \sin \theta$

14. Prove: $\frac{\sec \theta + \csc \theta}{1 + \tan \theta} = \csc \theta$

15. Solve: $2 \cos^2 \theta = 1 + \sin \theta$ for θ if $0 \leq \theta \leq 2\pi$.

16. Solve for x : $\sqrt{2} \cos x - 1 = 0$, $0 \leq x \leq 2\pi$.

16) _____

17. Solve a right triangle ABC, if $a = 6$ and $\angle A = 30^\circ$. (Give exact lengths.)

18. A boy flying a kite is standing 20 ft from a point directly under the kite. If the string to the kite is 40 ft long, find the angle of elevation of the kite.

19. An airplane is at an elevation of 30,000 ft and approaches the airport with an angle of descent of 5° . What is the distance between the airport and the point on the ground directly below the airplane?

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Answers

12a. 2

12b. $\frac{2\pi}{3}$

12c. $\frac{\pi}{2}$

12d. -4

13. $\cos^2 \theta$

14. answers will vary

15. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

16. $\frac{\pi}{4}, \frac{7\pi}{4}$

17. $\angle B = 60^\circ; b = 6\sqrt{3}; c = 12$

18. 60°

19. 342901.6 ft.

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20. Given $\triangle ABC$ with $a = 30$, $b = 20$, $c = 40$, find the largest angle.
21. The captain of a clipper ship spots two other ships on the ocean. One ship is 5 miles away while the other is 5.2 miles away. The angle between the two sightings is 20° . How far apart are the two observed ships? (to three decimals)
22. In $\triangle RST$, $\angle R = 137^\circ$, $t = 15$, and $s = 12$. Find r to the nearest integer.
23. If $\triangle ABC$ has $b = 30$, $\angle C = 40^\circ$ and $\angle A = 60^\circ$, find a to the nearest tenth.
24. Two angles of a triangle measure 29° and 51° . The longest side is 55cm. Find the length of the shortest side to the nearest tenth.
25. Solve for $\triangle ABC$ if $a = 15$, $c = 18$, and $\angle A = 32^\circ$.
- C = _____
- B = _____
- b = _____
- OR (if two triangles)*
- C = _____
- B = _____
- b = _____
26. Find the area of $\triangle ABC$ if $b = 32$, $c = 27$, and $\angle A = 108^\circ$.
27. The area of $\triangle PQR$ is 15. If $p = 5$ and $q = 10$, find all possible measures of $\angle R$.

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Answers

20. 104.5°

21. 1.782 miles

22. 25

23. 26.4

24. 27.1

25. $\angle C = 39^\circ$; $\angle B = 109^\circ$; $b = 27.0$ or $\angle C = 141^\circ$; $\angle B = 7^\circ$; $b = 3.5$

26. 410.86

27. 36.9° or 143.1°

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28. Find the area of a regular pentagon inscribed in a circle with radius 15.
29. The sides of an isosceles triangle have lengths 7, 10 and 10. What are the measures of its angles?
30. At a distance of 200 meters, the angle of elevation to the top of a building is 70° . Approximately how tall is the building?
31. Two ships leave a port on courses that differ by 70° and each travels at 25 knots. In terms of nautical miles, how far apart are the ships after 1 hour?
32. After leaving an airport, a plane flies for 1.75 hours at a speed of 200 k/h on a course of 100° . The plane then flies for 2 hours at a speed of 250 k/h on a course of 40° . At this time, how far from the airport is the plane?
33. Find the exact value of the following:
a. $\cos 75^\circ$ b. $\sin 105^\circ$
34. Simplify the following:
a. $\cos 75^\circ \cos 15^\circ - \sin 75^\circ \sin 15^\circ$ b. $\sin(30^\circ + x) + \sin(30^\circ - x)$
35. Suppose angle A is acute and $\cos A = \frac{5}{13}$. Find:
a. $\sin A$ b. $\cos 2A$ c. $\sin 2A$
36. Simplify the following:
a. $\frac{1 + \cos 2x}{\sin 2x}$ b. $(1 + \cot^2 x)(\cos 2x + 1)$ c. $\frac{\tan x}{\sec x - 1}$

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Answers

28. 534.97

29. $69.5^\circ, 69.5^\circ, 41^\circ$

30. 549.5 m

31. 28.7

32. 739.93 miles

33a. $\frac{\sqrt{6} - \sqrt{2}}{4}$

33b. $\frac{\sqrt{6} + \sqrt{2}}{4}$

34a. $\cos 90^\circ = 0$

34b. $\cos x$

35a. $\frac{12}{13}$

35b. $-\frac{119}{169}$

35c. $\frac{120}{169}$

36a. $\cot x$

36b. $2\cot^2 x$

36c. $\frac{\sin x}{1 - \cos x}$

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37. Evaluate the given expression: $1 - 2\sin^2 \frac{5\pi}{12}$

38. Prove: $(1 + \tan^2 x)(1 + \cos 2x) = 2$

39. Solve the following for $0 \leq x < 2\pi$.

a. $\cos 2x = \sin x - 2$ b. $2\sin^2 x = 3\cos x + 3$

c. $\sin x \tan x = 2\sin x$

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37. $-\frac{\sqrt{3}}{2}$

38. answers will vary

39a. $\frac{\pi}{2}$

39b. $\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

39c. $0\pi, \pi, 1.11, 4.25$