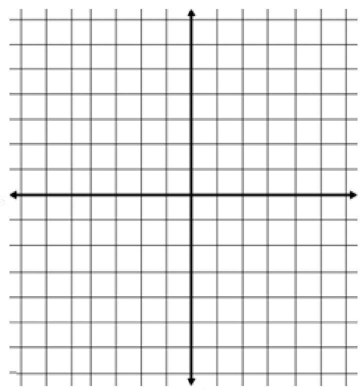
Factoring

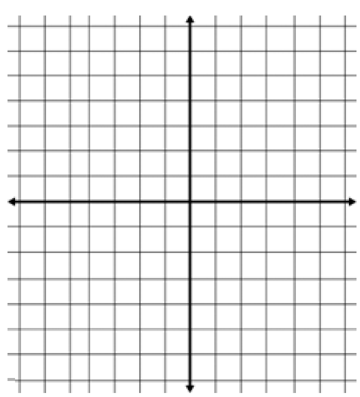
Factoring and Quadratic Functions

1. **Sketch the graph of the quadratic function. Identify the vertex, axis of symmetry, and x-intercept(s), domain, range and state the maximum or minimum value.**

(a) $f(x) = x^2 + 3x + 2$



(b) $f(x) = (x + 4)^2 - 3$



Answers

Factoring and Quadratic Functions

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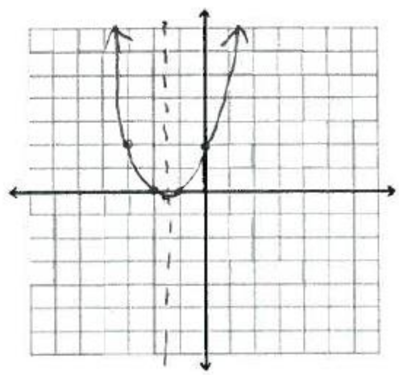


$f(x) = (x+2)(x+1)$

x-int @ $x = -2$, $x = -1$

$(-2, 0)\ (-1, 0)$

y-int @ $y = 2 \rightarrow (0, 2)$

Vertex @ $(-1.5, -0.25)$

$(-3/2, -1/4)$

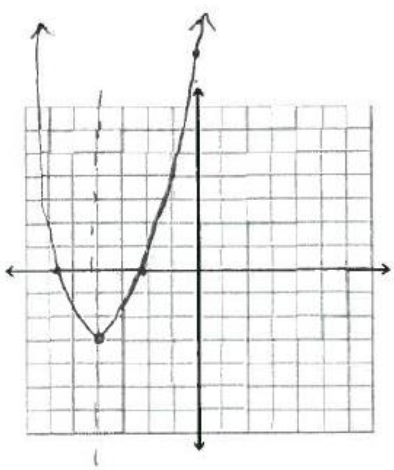
Domain: $x \in \mathbb{R}$

Range: $y \geq -1/4$

Min value: $y = -1/4$

Axis of symmetry: $x = -3/2$

(b) $f(x) = (x+4)^2 - 3$



Vertex: $(-4, -3)$

y-int: $y = 13$ $(0, 13)$

x-int: $(-2.27, 0)\ (-5.73, 0)$

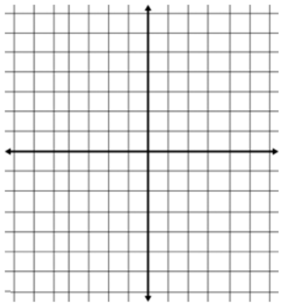
Axis of symmetry $x = -4$

Min value: $y = -3$

Domain: $x \in \mathbb{R}$

Range: $y \geq -3$

(c) $m(x) = -\frac{1}{3}x^2 + 3x - 6$



2. Write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

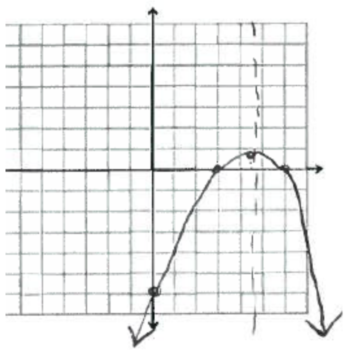
a) Vertex: (-2, 5); Point: (0, 9)

b) Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; x-intercept = -2

3. Write the standard form of the quadratic function that passes through the following 3 points. (0,2),(6,2)(-8,4)

Answers

(c) $m(x) = -\frac{1}{3}x^2 + 3x - 6$



y-int: $(0,-6)$
x-int: $(3,0)$ & $(6,0)$
Vertex $(9/2, 3/4)$
Max Value: $y = 3/4$
Axis of Symmetry: $x = 9/2$
Domain: $x \in \mathbb{R}$
Range: $y \le 3/4$

2. Write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

a) Vertex: (-2, 5); Point: (0, 9)

$f(x) = a(x+2)^2 + 5$

$9 = a(0+2)^2 + 5$

$9 = 4a + 5$

$4 = 4a$

$a = 1$

$\boxed{f(x) = 1(x+2)^2 + 5}$

b) Vertex: $\left(\frac{5}{2}, -\frac{3}{4}\right)$; x-intercept = -2 → $(-2, 0)$

$f(x) = a(x - 5/2)^2 - 3/4$ → Plug in $(-2, 0)$ $\quad 0 = a(-2 - \frac{5}{2})^2 - 3/4$

$3/4 = 20.25a \to a = 1/27$

$\boxed{f(x) = 1/27(x - 5/2)^2 - 3/4}$

3. Write the standard form of the quadratic function that passes through the following 3 points. (0,2),(6,2)(-8,4)

axis of symmetry $= \frac{6-0}{2} = 3$

vertex $= (3, k)$

$\boxed{f(x) = \frac{1}{56}(x-3)^2 + \frac{103}{56}}$

$f(x) = a(x-3)^2 + k$

$2 = a(-3)^2 + k$

$k = 2 - 9a \quad k = 103/56$

→ $f(x) = a(x-3)^2 + 2 - 9a$

$4 = a(-8-3)^2 + 2 - 9a$

$4 = 112a + 2 \quad a = 1/56$

4. **What is the maximum area of a rectangle that can be constructed with a perimeter of 64 cm?**

Solving Quadratic Equations

1) Solve the following Quadratic Equations (use the method of your choice)

a. $(x + 13)^2 = 25$

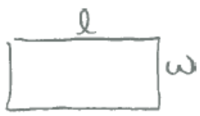
b. $(2x + 3)^2 - 27 = 0$

c. $(x - 7)^2 = (x + 3)^2$

Answers

4. **What is the maximum area of a rectangle that can be constructed with a perimeter of 64 cm?**

13



$2l + 2w = 64$

$l = \frac{64 - 2w}{2} \rightarrow l = 32 - w$

Area $\rightarrow A(l) = w(32 - w)$

$A(l) = -w^2 + 32w$

$A(l) = -1(w^2 - 32w + 256) + 256$

$= -1(w - 16)^2 + 256$ ← max area occurs when $w = 16$

$\boxed{\text{The maximum are is } 256\,cm^2}$

UNIT 5: Solving Quadratic Equations (CH. 6)

1) Solve the following Quadratic Equations (use the method of your choice)

a. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = 5 - 13 = -8$

$x = -5 - 13 = -18$

$\boxed{x = -8, -18}$

b. $(2x + 3)^2 - 27 = 0$

$(2x + 3)^2 = 27$

$2x + 3 = \pm\sqrt{27}$

$2x + 3 = \pm 3\sqrt{3}$

$2x = -3 \pm 3\sqrt{3}$

x

c. $(x - 7)^2 = (x + 3)^2$

d. $\frac{1}{8}x^2 - x - 16 = 0$

e. $3x^2 + 24x + 16 = 0$

f. $\frac{1}{4}x^2 - 2x + 7 = 0$

g. $12x - 9x^2 = -3$

h. $25x^2 + 80x + 61 = 0$

Answers

d. $\frac{1}{8}x^2 - x - 16 = 0$

$\frac{1}{8}(x^2 - 8x + \_\_) - 16 = 0$

$\frac{1}{8}(x^2 - 8x + 16) - 16 - 2 = 0$

$\frac{1}{8}(x-4)^2 - 18 = 0$

$(x-4)^2 = 144$

$\sqrt{(x-4)^2} = \pm\sqrt{144}$

$x - 4 = \pm 12$

$x = 4 \pm 12$

$\boxed{x = 16, -8}$

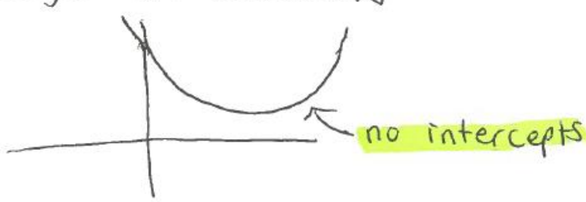
e. $3x^2 + 24x + 16 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-24 \pm \sqrt{24^2 - 4(3)(16)}}{2(3)} = \frac{-24 \pm \sqrt{384}}{6} = \frac{-24 \pm 8\sqrt{6}}{6}$

$\boxed{x = -0.73, -7.3}$ ← OR → $\boxed{= \frac{-12 \pm 4\sqrt{6}}{3}}$

f. $\frac{1}{4}x^2 - 2x + 7 = 0$

Solve by graphing: NO SOLUTION



g. $12x - 9x^2 = -3$

$-9x^2 + 12x + 3 = 0$

$-3(3x^2 + 4x + 1) = 0$

$-3(3x+1)(x+1) = 0$

$\boxed{x = -\frac{1}{3}, -1}$

h. $25x^2 + 80x + 61 = 0$

$x = \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)} = \frac{-80 \pm \sqrt{300}}{50} = \frac{-80 \pm 10\sqrt{3}}{50}$

$\boxed{x = \frac{-8 \pm \sqrt{3}}{5} = -1.25, -1.94}$

Factoring

1. $3x + 4 = 2x^2 - 7$

a. $2x^2 - 3x = 4x + 12$

2. Brian decides to start training for swimming in a river. The current in the river is 4km/hr. If he swims upstream 2 km and then back downstream to where he started in 3 hours, what is his swimming speed?

Factoring

Answers

i. $3x + 4 = 2x^2 - 7$

$2x^2 - 3x - 11 = 0$

* Solved by graphing

$\boxed{x = -1.71,\ 3.21}$

a. $2x^2 - 3x = 4x + 12$

$2x^2 - 7x - 12 = 0$

$\boxed{x = 4.76,\ -1.26}$

2. Brian decides to start training for swimming in a river. The current in the river is 4km/hr. If he swims upstream 2 km and then back downstream to where he started in 3 hours, what is his swimming speed?

$x =$ Brian's swimming speed

	R	D	t
Up	$x-4$	2km	$\frac{2}{x-4}$
Down	$x+4$	2km	$\frac{2}{x+4}$

total time $= 3 =$ time$_{up}$ + time$_{down}$

$$\frac{2}{x+4} + \frac{2}{x-4} = 3$$

$$2(x-4) + 2(x+4) = 3(x-4)(x+4)$$

$$2x - 8 + 2x + 8 = 3(x^2 - 16)$$

$$4x = 3x^2 - 48$$

$$3x^2 - 4x - 48 = 0$$

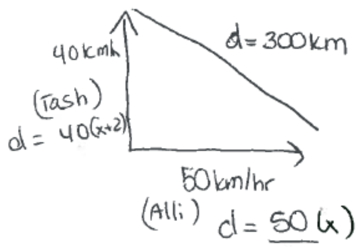
$x = 4.7$ km/hr

Brian's swimming speed is 4.7 km/hr

3. Natasha leaves school at 3pm and she drives north at 40 km/hr. 2 hours later (at 5pm), Alli leaves and she drives East at 50 km/hr. How long does it take before the two cars are 300 km apart?

Answers

3. Natasha leaves school at 3pm and she drives north at 40 km/hr. 2 hours later (at 5pm), Alli leaves and she drives East at 50 km/hr. How long does it take before the two cars are 300 km apart?



$t_{alli} = x$

$t_{tasha} = x + 2$

Pythagoras theorem

$(40(x+2))^2 + (50x)^2 = 300^2$

$(40x+80)^2 + 2500x^2 = 90000$

$1600x^2 + 6400x + 6400 + 2500x^2 = 90000$

$4100x^2 + 6400x - 83600 = 0$

$41x^2 + 64x - 836 = 0$

$x = 3.80$ hrs

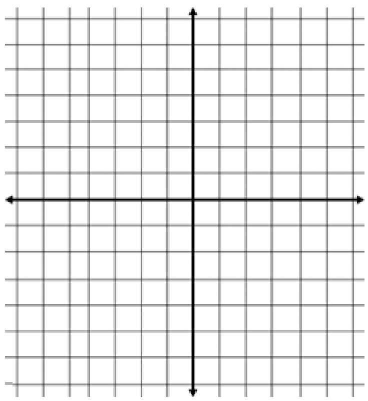
total time = tashas time = 5.8 hrs

It takes 5.8 hrs for the cars to be 300km apart

Factoring

1. Solve the system of equations and inequalities by graphing. If doing on calculator, sketch an accurate graph.

a) $\begin{cases} y = x^2 - 2x - 4 \\ y = -\frac{3}{4}x^2 - 4x + 4 \end{cases}$

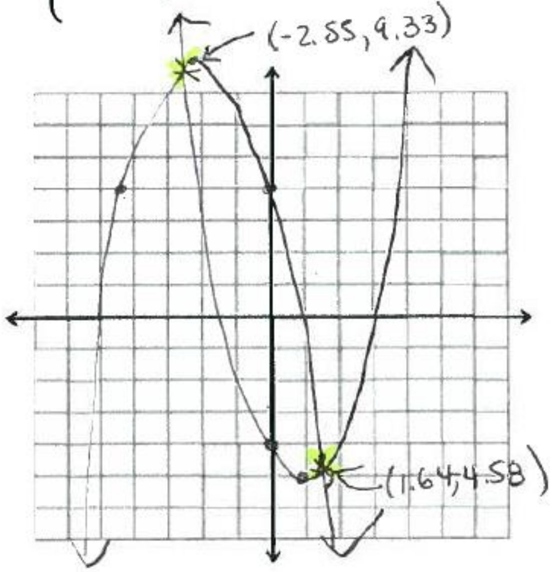


Factoring

Answers

1. Solve the system of equations and inequalities by graphing. If doing on calculator, sketch an accurate graph.

a) $\begin{cases} y = x^2 - 2x - 4 \\ y = -\frac{3}{4}x^2 - 4x + 4 \end{cases}$

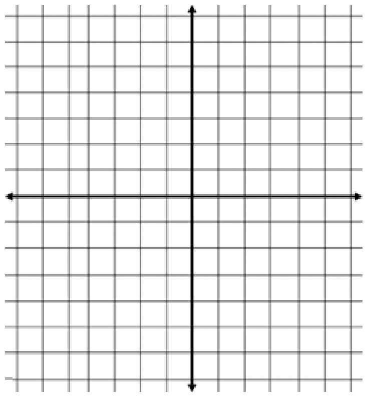


(-2.55, 9.32)

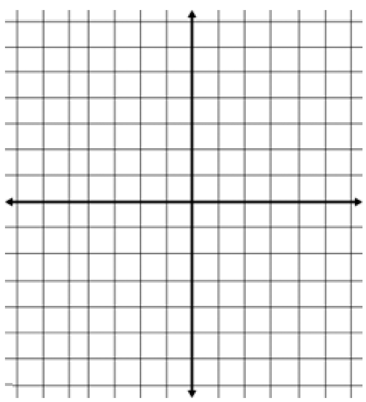
(1.64, -4.58)

Factoring

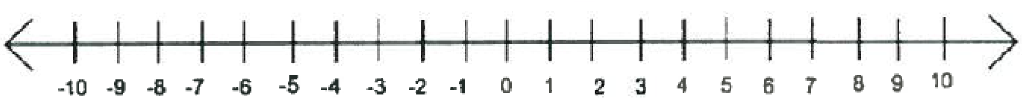
b) $\begin{cases} x^2 + y \leq 7 \\ x \geq -2 \\ y \geq 0 \end{cases}$



c) $\begin{cases} 2x + y > 2 \\ 6x + 3y < 2 \end{cases}$



d) $-2x^2 + 3x + 4 \geq 0$



Answers

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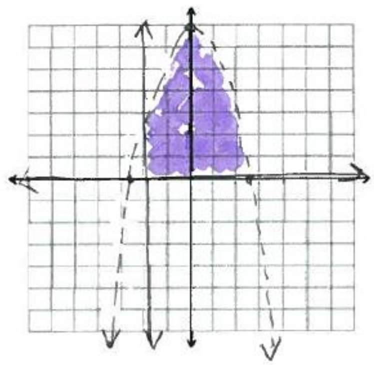
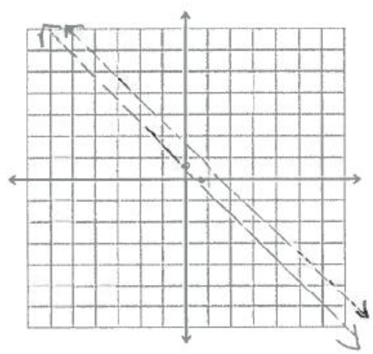


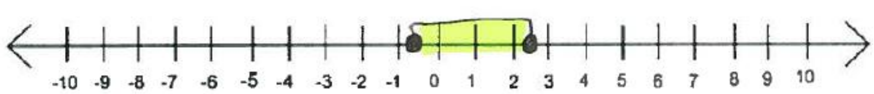
c) $\begin{cases} 2x + y > 2 \\ 6x + 3y < 2 \end{cases}$ $\quad y < \frac{2}{3} - \frac{6x}{3}$



NO SOLUTION

d) $-2x^2 + 3x + 4 \ge 0$

$$x = \frac{-3 \pm \sqrt{9 - 4(4)(-2)}}{2(-2)} = \frac{-3 \pm \sqrt{41}}{-4} = -0.85 \text{ or } 2.35$$



Check $x=0$ ✓ $4 \ge 0$ yes

* NOTICE FILLED IN CIRCLE

$$\boxed{-0.85 \le x \le 2.35}$$

2. Solve the following systems algebraically:

a) $\begin{cases} y = -x^2 - 4x + 5 \\ y = -3x + 7 \end{cases}$

b) $\begin{cases} -7x + 6y = -4 \\ 14x - 12y = 8 \end{cases}$

a) The profit for a construction company is $P(x) = -0.1x^2 + 50x - 5250$, where x is the total number of hours worked by the employees in a week. What total hours worked by the employees will produce a profit for the company?

Factoring

Answers

2. Solve the following systems algebraically:

a) $\begin{cases} y = -x^2 - 4x + 5 \\ y = -3x + 7 \end{cases}$

$-x^2 - 4x + 5 = -3x + 7$

$x^2 + x + 2 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(1)(2)}}{2(1)}$ ← NO SOLUTION

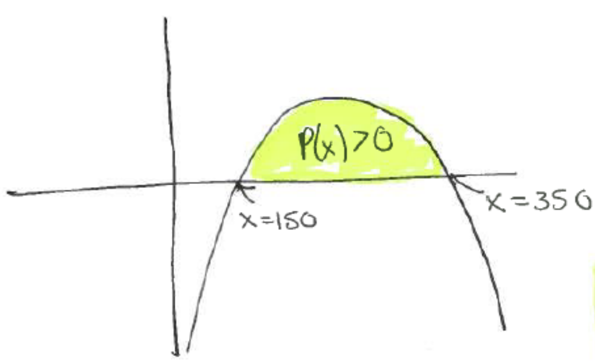
b) $\begin{cases} (-7x + 6y = -4)\times 2 \\ 14x - 12y = 8 \end{cases}$

$\left.\begin{aligned} -14x + 12y &= -8 \\ 14x - 12y &= 8 \end{aligned}\right\}$ Infinite # of solutions (same line)

a) The profit for a construction company is $P(x) = -0.1x^2 + 50x - 5250$, where x is the total number of hours worked by the employees in a week. What total hours worked by the employees will produce a profit for the company?

Graphing calculator $P(x) > 0$

$-0.1x^2 + 50x - 5250 > 0$



$\boxed{150 < x < 350}$

The construction company makes a profit when the employees work between 150 and 350 hours

b) The height in metres of a ball thrown upward from a building is $h(t)=-4.9t^2+29.4t+24.3$, where "t" is the time in seconds after releasing the ball. During what time interval will the ball be above 30 meters?

Factoring

Answers

b) The height in metres of a ball thrown upward from a building is $h(t)=-4.9t^2+29.4t+24.3$, where "t" is the time in seconds after releasing the ball. During what time interval will the ball be above 30 meters? Graphing calculator

$30 < -4.9t^2 + 29.4t + 24.3$

$0 < -4.9t^2 + 29.4t - 5.7 \rightarrow 0.2 < t < 5.8$

the ball will be above 30 meters between 0.2 and 5.8 seconds