

Absolute Value and Rational Functions

Absolute Value, Rational and Reciprocal Functions

1. Write the absolute value function as a piecewise function:

a) $g(x) = -4|x + 2| + 3$

b) $f(x) = \frac{1}{3}|2x - 7| + 9$

Absolute Value and Rational Functions

Answers

Absolute Value, Rational and Reciprocal Functions

1. Write the absolute value function as a piecewise function:

a) $g(x) = -4|x + 2| + 3$

$$g(x) = \begin{cases} -4(x+2)+3, & x \geq -2 \\ +4(x+2)+3, & x < -2 \end{cases}$$

$$g(x) = \begin{cases} -4x - 5, & x \geq -2 \\ 4x + 11, & x < -2 \end{cases}$$

b) $f(x) = \frac{1}{3}|2x - 7| + 9$

$$f(x) = \begin{cases} \frac{1}{3}(2x-7)+9, & x \geq \frac{7}{2} \\ -\frac{1}{3}(2x-7)+9, & x < \frac{7}{2} \end{cases}$$

$$\begin{cases} 2x-7 \geq 0 \\ 2x \geq 7 \\ x \geq \frac{7}{2} \end{cases} \quad \begin{cases} 2x-7 < 0 \\ x < \frac{7}{2} \end{cases}$$

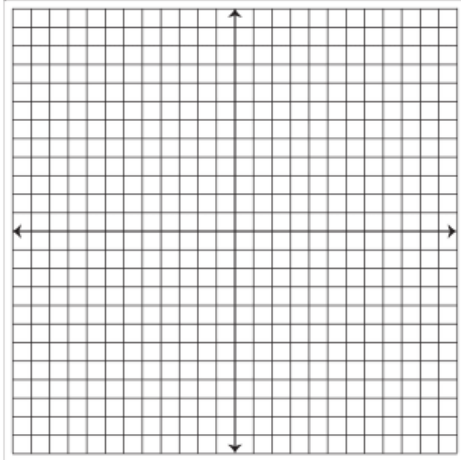
$$f(x) = \begin{cases} \frac{2x}{3} - \frac{7}{3} + \frac{27}{3}, & x \geq \frac{7}{2} \\ -\frac{2x}{3} + \frac{7}{3} + \frac{27}{3}, & x < \frac{7}{2} \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{2x}{3} + \frac{20}{3}, & x \geq \frac{7}{2} \\ -\frac{2x}{3} + \frac{34}{3}, & x < \frac{7}{2} \end{cases}$$

Absolute Value and Rational Functions

2. Graph the absolute value function. State the vertex, intercepts (x and y), domain and range.

a) $h(x) = -\frac{1}{2}|x - 4| + 4$



3. Solve the following absolute value functions:

a) $|x+3|=-3x$

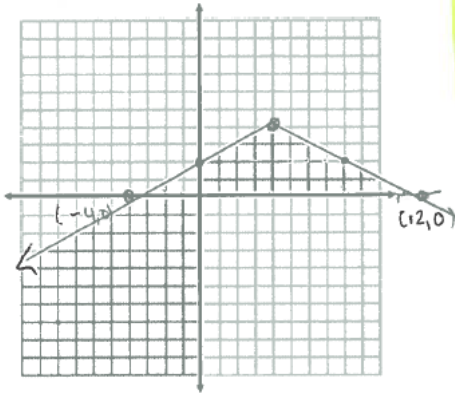
b) $|x-3|=|2x+4|$

Absolute Value and Rational Functions

Answers

2. Graph the absolute value function. State the vertex, intercepts (x and y), domain and range.

a) $h(x) = -\frac{1}{2}|x - 4| + 4$



Vertex (4, 4)
 y-intercept (0, 2)
 x-intercept (-4, 0) (12, 0)
 Domain: $x \in \mathbb{R}$
 Range: $y \leq 4$

3. Solve the following absolute value functions:

a) $|x+3| = -3x$

① $x+3 = -3x$

$3 = -4x$

$x = \frac{3}{-4}$

✓
 yes
 $2.25 = 2.25$

② $x+3 = 3x$

$3 = 2x$

$x = \frac{3}{2}$

X No → extraneous

$|3\frac{1}{2} + 3| \neq -3(3\frac{1}{2})$

$4.5 \neq -4.5$

ONLY $x = -\frac{3}{4}$

b) $|x-3| = |2x+4|$

case ① → $x-3 = 2x+4$

$-7 = x$

check: $|-10| = |-10|$ ✓ yes

case ② → $-x+3 = 2x+4$

$-1 = 3x$

$x = -\frac{1}{3}$

check: $|\frac{10}{3}| = |\frac{10}{3}|$ → yes

case ③ → $x-3 = -2x-4$

$3x = -1$

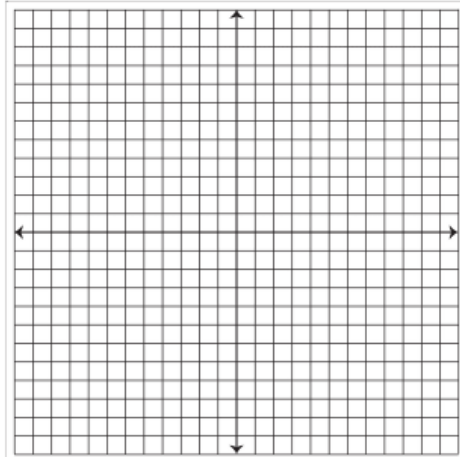
$x = -\frac{1}{3}$ → yes!

$x = -7, -\frac{1}{3}$

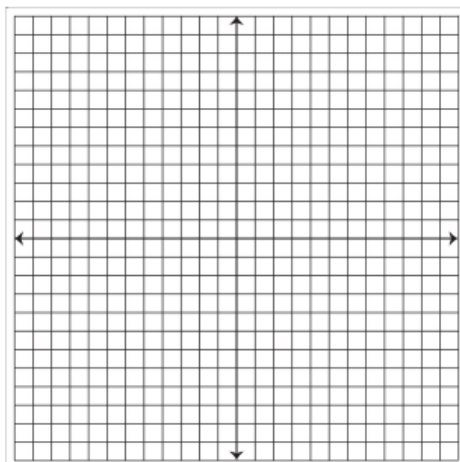
Absolute Value and Rational Functions

4. Sketch the following graphs and label both the horizontal and vertical asymptotes. State the domain of each function.

a) $f(x) = \frac{3x}{x-1}$



b) $h(x) = \frac{x^2-1}{x^2+2x-3}$

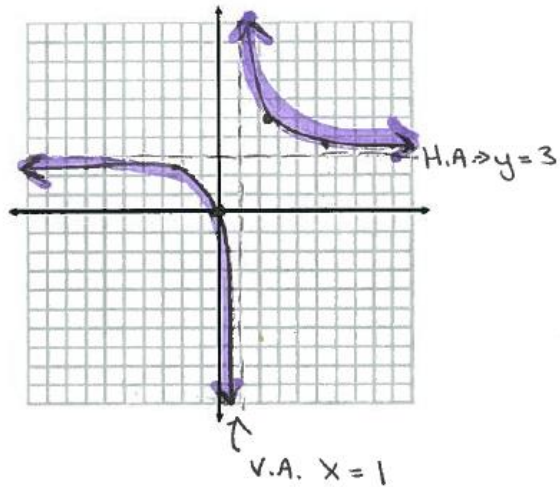


Absolute Value and Rational Functions

Answers

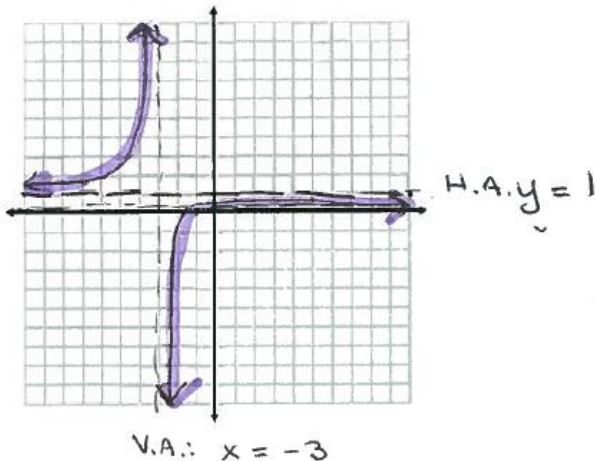
4. Sketch the following graphs and label both the horizontal and vertical asymptotes. State the domain of each function.

a) $f(x) = \frac{3x}{x-1}$



V.A. $\Rightarrow x = 1$
H.A. $\Rightarrow y = 3$

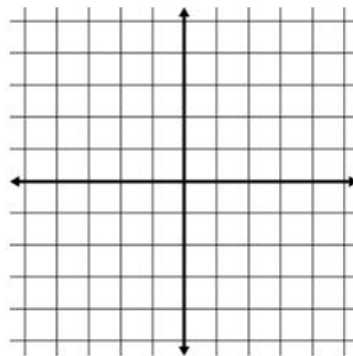
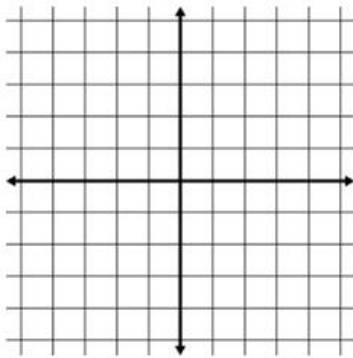
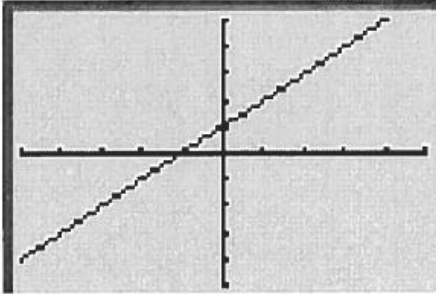
b) $h(x) = \frac{x^2-1}{x^2+2x-3} = \frac{(x-1)(x+1)}{(x+3)(x-1)} = \frac{x+1}{x+3}$ H.A. $\rightarrow y = 1$
V.A. $\rightarrow x = -3$



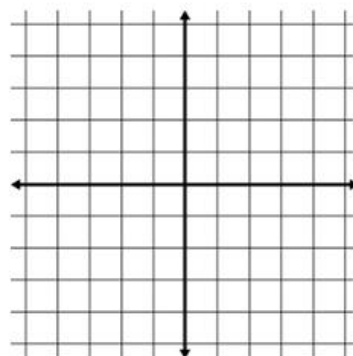
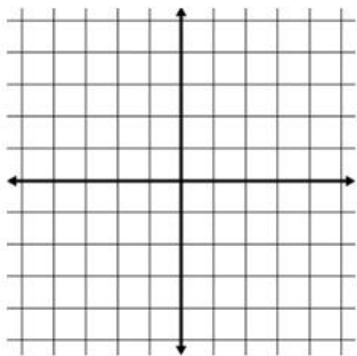
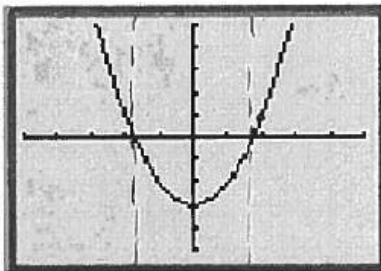
Absolute Value and Rational Functions

5. Given the following graph of $y = f(x)$, sketch the graph of $y = \frac{1}{f(x)}$ and the graph of $y = |f(x)|$

a)



b)

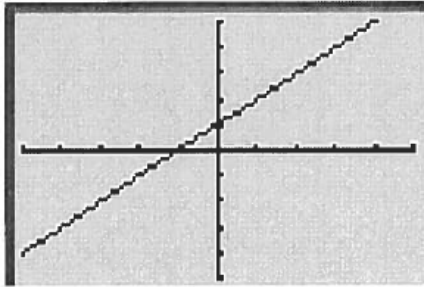


Absolute Value and Rational Functions

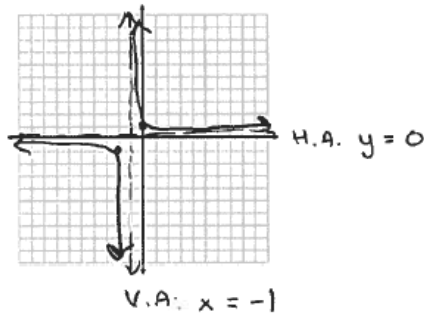
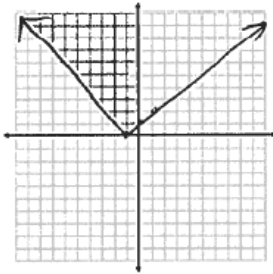
Answers

5. Given the following graph of $y = f(x)$, sketch the graph of $y = \frac{1}{f(x)}$ and the graph of $y = |f(x)|$

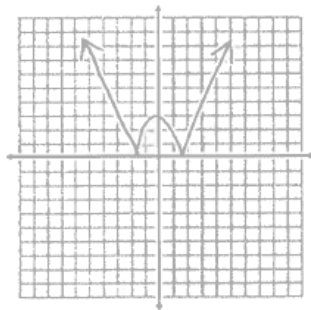
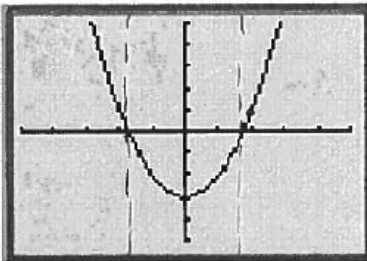
a)



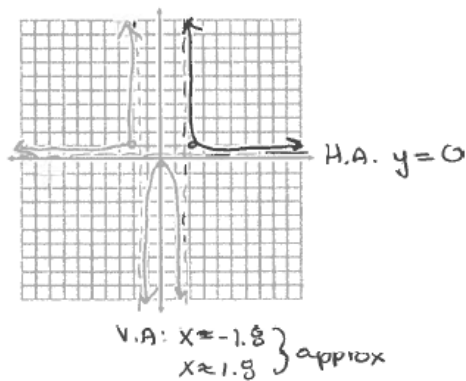
$y = |f(x)|$



b)



$y = |f(x)|$

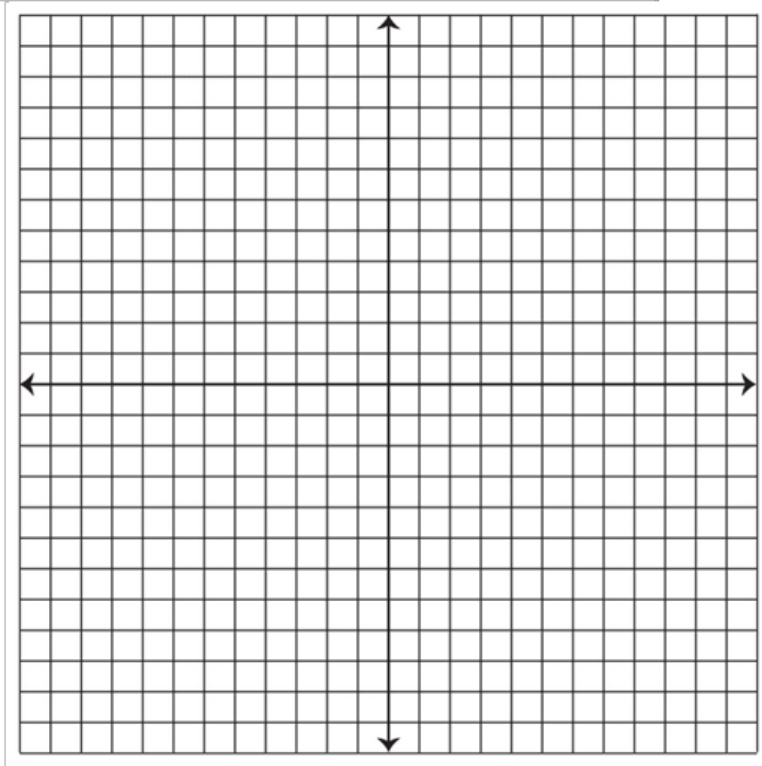


Absolute Value and Rational Functions

6. Sketch a graph of a rational function that has the following characteristics.

a)

x-intercept	-1
y-intercept	-2
Vertical Asymptote	$x=-2.5$
Horizontal Asymptote	$y=-3.5$



Absolute Value and Rational Functions

Answers

6. Sketch a graph of a rational function that has the following characteristics.

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x-intercept	-1
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