## Sequences and Series

1. Given the series defined by $\sum_{k=2}^{15} 16\left(\frac{1}{2}\right)^{k-1}$, determine the common ratio, the number of terms and the sum.

Answers

**Sequences and Series**

**1. Given the series defined by $\sum_{k=2}^{15} 16\left(\frac{1}{2}\right)^{k-1}$, determine the common ratio, the number of terms and the sum.**

$\underset{(k=2)}{a_1} = 16\left(\frac{1}{2}\right)^1$

$a_1 = 8$

$\underset{(k=3)}{a_2} = 16\left(\frac{1}{2}\right)^{3-1}$

$= 16\left(\frac{1}{4}\right) = 4$

$r = \frac{a_2}{a_1} = \frac{4}{8} = 1/2$

number of terms $= 15 - 2 + 1 = 14$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{8\left(1-\left(\frac{1}{2}\right)^{14}\right)}{1 - 1/2} =$

$\boxed{S_{14} = 15.999}$

**2. Find the $14^{th}$ term of the sequence {3,11,19….}**

**3. Three consecutive terms of a geometric sequence are 2.5, y+3 and 9.6. Find the value of y.**

**4. Compute the sum of the first 8 terms in the sequence {1,-3, 9…..}**

**5. How can you tell whether or not an infinite geometric series has a finite or an infinite sum?**

**6. Find the $18^{th}$ term in an arithmetic sequence who's $2^{nd}$ term is 11 and who's $8^{th}$ term is 41.**

# Sequences and Series

## Answers

2. **Find the 14th term of the sequence {3,11,19….}**

$d = 8$
$a = 1$
$n = 14$

$t_{14} = a + (n-1)d$
$t_{14} = 1 + (14-1)8$
$t_{14} = 1 + 13 \times 8$

$\boxed{t_{14} = 105}$

3. **Three consecutive terms of a geometric sequence are 2.5, y+3 and 9.6. Find the value of y.**

$\frac{y+3}{2.5} = \frac{9.6}{y+3}$

$(y+3)(y+3) = (9.6)(2.5)$

$y^2 + 6y + 9 = (9.6)(2.5)$
$y^2 + 6y + 9 = 24$
$y^2 + 6y - 15 = 0$ → solved by graphing / reject −ve ans

$\boxed{y = 1.9 \text{ OR } -7.9}$

4. **Compute the sum of the first 8 terms in the sequence {1,-3, 9…..}**

change*

$r = \frac{-3}{1} = -3$

$n = 8$
$a = 1$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1(1-(-3)^8)}{1-(-3)}$

$\boxed{S_8 = -1640}$

5. **How can you tell whether or not an infinite geometric series has a finite or an infinite sum?**

$|r| < 1 \rightarrow \boxed{-1 < r < 1}$ and $\boxed{r \neq 0}$ } The common difference must be between −1 and 1 and cannot = 0

6. **Find the 18th term in an arithmetic sequence who's 2nd term is 11 and who's 8th term is 41.**

$t_2 = 11 = a + (2-1)d$
$t_8 = 41 = a + (8-1)d$

$\rightarrow 11 = a + d$
$-(41 = a + 7d)$
$-30 = -6d$

$d = 5$
$a = 6$ } plug in to find $t_{18}$

$t_{18} = 6 + (18-1)5$
$t_{18} = 6 + 17(5)$
$\boxed{t_{18} = 91}$

7. Find the 9th term in a geometric sequence who's first term is 6 and who's 4th term is -3/4.

8. Calculate the sum of the infinite geometric series given by

$$\sum_{k=2}^{\infty} 8\left(-\frac{1}{2}\right)^{k-1}$$

Answers

7. **Find the 9$^{th}$ term in a geometric sequence who's first term is 6 and who's 4$^{th}$ term is -3/4.**

$t_1 = a = 6$

$t_4 = ar^{n-1}$

$t_4 = 6(r)^{4-1} \rightarrow * t_4 = -3/4$

$-3/4 = 6(r)^3$

$-\frac{3}{24} = r^3$ . $-\frac{1}{8} = r^3 \rightarrow \boxed{r = \frac{-1}{2}}$

$t_9 = 6\left(\frac{-1}{2}\right)^{9-1}$

$\boxed{t_9 = \frac{3}{128}}$

8. **Calculate the sum of the infinite geometric series given by**

$$\sum_{k=2}^{\infty} 8\left(-\frac{1}{2}\right)^{k-1}$$

$a = 8\left(\frac{-1}{2}\right)^{2-1}$ $(k=2)$

$= 8\left(\frac{-1}{2}\right) = -4$

$a_2 = 8\left(\frac{-1}{2}\right)^{3-1} = 8\left(\frac{1}{4}\right) = 2$ $(k=3)$

$r = \frac{2}{-4} = \frac{-1}{2}$

$S_\infty = \frac{a}{1-r} = \frac{-4}{1-\left(\frac{-1}{2}\right)} = -1.6$

$$\boxed{S_\infty = -\frac{8}{5}}$$