1. Simplify each of the following expressions.

a) $\dfrac{3^{2x+1}}{9^{x-1}}$ b) $\dfrac{\left(8^{b-2}\right)\left(2^{b+1}\right)}{4^{2b-3}}$ c) $5^{2x-3}\cdot 25^{1-x}$ d) $\sqrt{\dfrac{2^{3x-1}\cdot 3^{2-x}}{6^{x+1}}}$

2. Solve each of the following equations.

a) $5^{x-2}=4$

b) $5e^{-3x}=42$

c) $2^{2x-3}=5^{2-x}$

d) $5\cdot 3^{2x-1}=2^{2-x}$

e) $3\cdot 2^{2x-3}=2\cdot 5^{x+2}$

f) $9^x+4\cdot 3^x=5$

g) $e^{2x}-7e^x+10=0$

h) $4^x-2^{x+2}=-3$

i) $9^x-3^{x+1}=4$

j*) $9^x-4\cdot 6^x+3\cdot 4^x=0$

k*) $4^x+200\cdot 25^x=30\cdot 10^x$

3. We placed \$1000 in a bank account with an annual compound interest rate of 8%. How long will it take until we have \$5000 in the account?

4. If we take Q amount of a certain medication, the amount of it in our system, t hours after intake is

$$A(t)=Q\cdot 0.8^t$$

a) Approximately what percent of the mediaction is in our system 2 hours after taking it?
b) How long until we have only 20% left in our system?
c) How long until we have only 1% left in our system?

5. The number of cells in a sample at time t (measured in hours) is $N(t)=50\,000\ \left(1.6^{0.5t}\right)$.
a) How many cells are there at $t=0$?
b) How long will it take for the sample to double from the time $t=0$?
c) How many cells are there at $t=4$?
d) How long will it take for the sample to double from the time $t=4$?
e) Suppose that t_1 and t_2 are given such that $N(t_2)=2N(t_1)$. Prove that the difference t_2-t_1 is constant.

Answers

1. a) 27 b) 2 c) $\frac{1}{5}$ d) $\frac{\sqrt{3}}{2}\left(\frac{2}{3}\right)^x$

2. a) $x = \log_5 100 = \frac{\ln 4}{\ln 5} + 2$ b) $x = \frac{1}{3}\ln\left(\frac{5}{42}\right) = \frac{\ln 8.4}{-3}$ c) $x = \log_{20} 200 = \frac{\ln 200}{\ln 20}$

   d) $\log_{18}\left(\frac{12}{5}\right) = \frac{\ln\left(\frac{12}{5}\right)}{\ln 18}$ e) $\log_{4/5}\left(\frac{400}{3}\right) = \frac{\ln\left(\frac{400}{3}\right)}{\ln\left(\frac{4}{5}\right)}$ f) $x = 0$ g) $\ln 2$ and $\ln 5$

   h) 0 and $\log_2 3$ i) $\log_3 4$ j*) $0, \frac{\ln 3}{\ln\left(\frac{3}{2}\right)}$ k*) $\log_{2/5} 20$ and $\log_{2/5} 10$.

3. during the 21st year $\left(x = \frac{\ln 5}{\ln 1.08} \approx 20.912\,371\,88\right)$

4. a) 64% b) $\frac{\ln 0.2}{\ln 0.8} \approx 7.212\,567$ hours c) $\frac{\ln 0.01}{\ln 0.8} \approx 20.637\,7$ hours

5. a) 50 000 b) $\frac{\ln 2}{0.5\ln 1.6} \approx 2.95$ hours c) 128 000 d) 2.95 hours e) see solutions

1. Simplify each of the following expressions so that there is at most one exponential expression is in the answer, with an exponent of x.

a) 2^{2x+3}

b) $5 \cdot 3^{2-x}$

c) $\dfrac{2^{x-2}}{5 \cdot 3^{2x+1}}$

d) $\dfrac{3 \cdot 5^{2x+1}}{2^{4-x}}$

e) $\dfrac{2^{x-1} \cdot 5^{x+2}}{10^{x-2}}$

f) $\dfrac{2^{2x+1} \cdot 3^{x-1}}{6^{x-1}}$

g) $\dfrac{5 \cdot 12^{m+1}}{3^{m-1} \cdot 2^{2m+1}}$

h) $\left(\dfrac{1}{5}\right)^{2p-3} 25^{p-1}$

i) $\sqrt{\dfrac{3^{x-1} \cdot 6^{x+2}}{2^{x-3}}}$

j) $\sqrt{2^{10x}} \cdot \left(\dfrac{1}{8}\right)^{x-2} \cdot 4^{-x-1}$

k) $\sqrt{\dfrac{2^{6x} \cdot 5^{8x-2}}{10^{6x+2}}}$

2. Solve each of the following equations.

a) $2 \cdot 3^{x-5} - 7 = 23$

b) $3e^{2x} - 8 = 13$

c) $4^x - 2^x - 12 = 0$

d) $\left(\dfrac{1}{9}\right)^x - \dfrac{6}{3^x} + 8 = 0$

e) $3^x + \dfrac{9}{3^x} = 10$

f*) $4^x - 7 \cdot 10^x + 10 \cdot 5^x = 0$

g) $5^{x-2} = 2^{2x+3}$

h*) $6 \cdot 4^x - 13 \cdot 6^x + 6 \cdot 9^x = 0$

i) $e^{2x} + e^x = 6$

j) $5 \cdot 2^{3x-1} = 3 \cdot 5^{2-x}$

k) $4 \cdot 3^{x-2} = 6^{x+1}$

l) $9^x - 3^{x+1} = 54$

m) $\dfrac{10^{x+2}}{2^{x-3}} = 5^{x+1}$

3. We placed \$50 in a bank account with an annual compound interest rate of 13%. How long will it take until the account contains

a) \$100 b) \$2500 c) \$1000 000

4. If we take Q amount of a certain medication, the amount of it in our system, t hours after intake is

$$A(t) = Q \cdot \left(\frac{7}{8}\right)^t$$

a) Approximately what percent of the mediaction is in our system 5 hours after taking it?
b) How long until we have 60% left in our system?
c) How long until we have only 1% left in our system?
d) How long does it take for the drug to reduce to half? (This is called the half-life of the drug.)

5. The number of cells in a sample at time t (measured in hours) is $N(t) = 100\,000 \left(1.4^{0.3t}\right)$.
a) How many cells are there at $t = 0$?
b) How long will it take for the sample to triple from the time $t = 0$?

Answers

1. a) $8 \cdot 4^x$ b) $\frac{45}{3^x}$ c) $\frac{1}{60}\left(\frac{2}{9}\right)^x$ d) $\left(\frac{15}{16}\right) \cdot 50^x$ e) 1250 f) $4 \cdot 2^x$ g) 90
   h) 5 i) $4\sqrt{6}\,(3^x)$ j) 16 k) $\frac{5^x}{50}$

2. a) $\frac{\ln 15}{\ln 3} + 5$ b) $\frac{1}{2}\ln 7$ c) 2 d) $-\frac{\ln 2}{\ln 3}, -\frac{\ln 4}{\ln 3}$ e) 0, 2 f) $\frac{\ln 2}{\ln 2 - \ln 5}, \frac{\ln 5}{\ln 2 - \ln 5}$
   g) $\frac{\ln 200}{\ln 5 - \ln 2}$ h) ± 1 i) $\ln 2$ j) $\log_{40} 30$ k) $\log_2\left(\frac{2}{27}\right)$ l) 2 m) no solution

3. a) during the 6th year $\left(x = \frac{\ln 2}{\ln 1.13} \approx 5.671\,42\right)$
   b) during the 33rd year $\left(x = \frac{\ln 50}{\ln 1.13} \approx 32.008\,663\right)$
   c) during the 82nd year $\left(x = \frac{\ln 20\,000}{\ln 1.13} \approx 81.031\,577\right)$

4. a) 51.2 909% b) $\frac{\ln 0.6}{\ln\left(\frac{7}{8}\right)}$ hours $\approx 3.825\,51$ hours c) $\frac{\ln 0.01}{\ln\left(\frac{7}{8}\right)}$ hours $\approx 34.487\,55$ hours
   d) $\frac{\ln 0.5}{\ln\left(\frac{7}{8}\right)} \approx 5.190\,9$ hours

5. a) 100 000 b) $\frac{\ln 3}{0.3 \ln 1.4}$ hours $\approx 10.883\,635$ hours