SOLVING LINEAR EQUATIONS

Goal: The goal of solving a linear equation is to find the value of the variable that will make the statement (equation) true.

Method: Perform operations to both sides of the equation in order to isolate the variable.

Addition and Subtraction Properties of Equality:

Let a, b, and c represent algebraic expressions.

1. **Addition property of equality:**

   If $\quad a = b$,

   then $a + c = b + c$

2. **Subtraction property of equality:**

   If $\quad a = b$,

   then $a - c = b - c$

Multiplication and Division Properties of Equality:

Let a, b, and c represent algebraic expressions.

1. **Multiplication property of equality:**

   If $\quad a = b$,

   then $ac = bc$

2. **Division property of equality:**

   If $\quad a = b$,

   then $\frac{a}{c} = \frac{b}{c}$ (provided $c \neq 0$)

Clearing Fractions or Decimals in an Equation:

When solving an equation with fractions or decimals, there is an option of clearing the fractions or decimals in order to create a simpler equation involving whole numbers.

1. To clear fractions, multiply both sides of the equation (distributing to all terms) by the LCD of all the fractions.
2. To clear decimals, multiply both sides of the equation (distributing to all terms) by the lowest power of 10 that will make *all* decimals whole numbers.

Steps for Solving a Linear Equation in One Variable:

1. Simplify both sides of the equation.
2. Use the addition or subtraction properties of equality to collect the variable terms on one side of the equation and the constant terms on the other.
3. Use the multiplication or division properties of equality to make the coefficient of the variable term equal to 1.
4. Check your answer by substituting your solution into the original equation.

Note: If when solving an equation, the variables are eliminated to reveal a *true* statement such as, $-13 = -13$, then the solution is **all real numbers.** This type of equation is called an **identity**. On the other hand, if the variables are eliminated to reveal a *false* statement such as, $-7 = 3$, then there is **no solution**. This type of equation is called a **contradiction**. All other linear equations which have only one solution are called **conditional**.

Examples:

A. $x + 5 = -2$

$\quad \underline{-5 \quad -5}$

$\qquad x = -7$

Check:

$(-7) + 5 = -2$

$\qquad -2 = -2$ (Solution Checks)

B. $x - 3 = 7$

$\quad \underline{+3 \ +3}$

$\qquad x = 10$

Check:

$10 - 3 = 7$

$\qquad 7 = 7$ (Solution Checks)

C. $3x = 24$

$\dfrac{3x}{3} = \dfrac{24}{3}$

$x = 8$

Check:

$3(8) = 24$

$24 = 24$ (Solution Checks)

D. $-7x = 28$

$\dfrac{-7x}{-7} = \dfrac{28}{-7}$

$x = -4$

Check:

$-7(-4) = 28$

$28 = 28$ (Solution Checks)

E. $\dfrac{2}{3}x = -4$

$\dfrac{3}{2} \cdot \dfrac{2}{3}x = \dfrac{-4}{1} \cdot \dfrac{3}{2}$ (Multiply by the reciprocal)

$x = -6$

Check:

$\dfrac{2}{3}\left(\dfrac{-6}{1}\right) = -4$

$-4 = -4$ (Solution Checks)

F. $\dfrac{y}{-5} = 2$

$\dfrac{-5}{1} \cdot \dfrac{y}{-5} = 2(-5)$

$y = -10$

Check:

$\dfrac{-10}{-5} = 2$

$2 = 2$ (Solution Checks)

Identity Example:

G. $2x + 6 = 3(x + 2) - x$

    $2x + 6 = 3x + 6 - x$

    $2x + 6 = 2x + 6$

  $\underline{-2x \qquad - 2x}$

           $6 = 6$ True Statement

Solution: all real numbers

I. $10 - 5x = 3(x - 4) - 2(x + 7)$

    $10 - 5x = 3x - 12 - 2x - 14$

    $10 - 5x = x - 26$

    $\underline{\qquad -x \ - x \qquad}$

    $10 - 6x = -26$

  $\underline{-10 \qquad\quad - 10}$

       $-6x = -36$

       $\dfrac{-6x}{-6} = \dfrac{-36}{-6}$

         $x = 6$

J. $\dfrac{x-2}{5} - \dfrac{x-4}{2} = 2$ (LCD is 10)

  $10\left(\dfrac{x-2}{5} - \dfrac{x-4}{2}\right) = 2(10)$

  $\dfrac{10}{1} \cdot \dfrac{(x-2)}{5} - \dfrac{10}{1} \cdot \dfrac{(x-4)}{2} = 20$

  $2(x - 2) - 5(x - 4) = 20$

  $2x - 4 - 5x + 20 = 20$

  $-3x + 16 = 20$

  $\underline{\qquad -16 - 16}$

      $\dfrac{-3x}{-3} = \dfrac{4}{-3}$

        $x = -\dfrac{4}{3}$

Contradiction Example:

H. $5x - 3 = 4(x + 2) + x$

$\quad 5x - 3 = 4x + 8 + x$

$\quad 5x - 3 = 5x + 8$

$\underline{-5x \quad\quad - 5x}$

$\quad\quad -3 = 8$ False Statement

No Solution

Check:

$10 - 5(6) = 3(6 - 4) - 2(6 + 7)$

$\quad 10 - 30 = 3(2) - 2(13)$

$\quad\quad -20 = 6 - 26$

$\quad\quad -20 = -20$ (Solution Checks)

K. $0.05x + 0.25 = 0.2$

$\quad 100(0.05x + 0.25) = 0.2(100)$

$\quad 5x + 25 = 20$

$\quad\underline{\quad -25 \; - 25}$

$\quad\quad \dfrac{5x}{5} = \dfrac{-5}{5}$

$\quad\quad x = -1$

Check:

$0.05(-1) + 0.25 = 0.2$

$-0.05 + 0.25 = 0.2$

$\quad\quad 0.2 = 0.2$ (Solution Checks)

LINEAR EQUATIONS PRACTICE

1. $4x = 4$

2. $x + 6 = -7$

3. $x - 4 = 7$

4. $\frac{x}{3} = -9$

5. $2x + 4 = 8$

6. $14 = 3 + 2x$

7. $8x - 3 = -19$

8. $6 - x = 9$

9. $-x = -12$

10. $3(x - 2) = 6$

11. $-3(2x - 8) = -12$

12. $4(6 + 2x) = 0$

13. $3x + 2x + 6 = -15$

14. $4 = -2(x + 3)$

15. $27 = 46 + 2x - x$

16. $4x + 6 - 7x + 9 = 18$

17. $4 + 3(x + 2) = 10$

18. $-3 + 3x = -2(x + 1)$

19. $9x - 6 = -3x + 30$

20. $-(x + 2) = 2(3x - 6)$

21. $2x + 6 = 3x + 9 - 3$

22. $-5x + 3 = 2x + 10$

23. $3x - 12x = 24 - 9x$

24. $2(x + 4) = -3(x + 5)$

25. $4(2x - 3) + 4 = 8x - 8$

26. $6x + 11 = -(6x + 5)$

27. $2(x + 7) = 6x + 9 - 4x$

28. $-5(3 - 4x) = -6 + 20x - 9$

29. $4(x - 3) - (x - 5) = 0$

30. $-2(4 - x) = 6(x + 2) + 3x$

31. $\frac{4}{7} = \frac{x}{21}$

32. $\frac{x}{4} = \frac{-20}{16}$

33. $\frac{9c}{10} = \frac{9}{5}$

34. $\frac{1}{4} = \frac{z+1}{4}$

35. $\frac{a}{5} = \frac{a-3}{2}$

36. $\frac{n}{10} = 9 - \frac{n}{5}$

37. $\frac{2}{8} + \frac{3}{4} = \frac{w}{5}$

38. $x - \frac{3}{4} = -2x$

39. $\frac{x}{4} - \frac{x}{6} = \frac{1}{4}$

40. $a - \frac{a}{3} + \frac{a}{5} = 26$

41. $\frac{12}{10} = \frac{z}{25}$

42. $\frac{-2}{6} = \frac{3c}{9}$

43. $\frac{x+4}{7} = \frac{3}{7}$

44. $\frac{4x+5}{6} = \frac{7}{2}$

45. $6 - \frac{x}{4} = \frac{x}{8}$

46. $\frac{x}{3} - \frac{3x}{4} = \frac{1}{12}$

47. $\frac{5}{2} - x = 3x$

48. $\frac{3-5y}{4} = \frac{2-4y}{3}$

49. $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

50. $-\frac{x}{4} = 12$

51. $-x = -12$

52. $-2x = -16$

53. $2x = -14$

54. $\frac{1}{7}x = -8$

55. $\frac{1}{7}x = 2$

56. $-\frac{x}{2} = 4$

57. $-x = 26$

58. $3x = 15$

59. $4x = -32$

60. $\frac{1}{3}x = 5$

61. $\frac{1}{9}x = 5$

62. $-\frac{5}{3}x = -15$

63. $-\frac{6}{5}x = -30$

64. $\frac{6}{5}x = 90$

65. $\frac{1}{3}x = 4$

66. $\frac{7}{6}x = 168$

67. $\frac{1}{6}x = 2$

68. $-\frac{9}{5}x = -45$

69. $-\frac{4}{9}x = -36$

70. $4x - 4 = -40$

71. $9x - 7 = -34$

72. $\frac{7}{8}y - 6 = 8$

73. $10 - x = 6$

74. $-2x - 4x = 1$

75. $-9x - 9x = -9$

76. $\frac{x}{3} - \frac{x}{5} = 2$

77. $\frac{x}{7} - \frac{x}{9} = 2$

78. $\frac{x}{3} - \frac{x}{9} = 6$

79. $\frac{x}{8} - \frac{x}{9} = 1$

80. $\frac{5}{9}y - 4 = 6$

81. $x + 0.4x = 3.5$

82. $5(x - 3) = 45$

83. $-3(x + 7) = 9$

84. $-4(x - 6) = 12$

85. $8 = 2(x - 5) + 6x$

86. $2 = 7(x + 4) + 9x$

87. $1 = 3(x - 2) + 3 - 2x$

88. $3 = 4(x - 2) + 5 - 3x$

89. $3.65 - 7.4x + 1.12 = 21.76$

90. $-8x + 3 - 2x = -6x + 3 - 4x$

91. $10x + 3 + 10x = 13x - 3 + 7x$

92. $6 + 3x = 5(x - 1) - 3(x - 2)$

93. $10 - 5x = 3(x - 4) - 2(x + 7)$

94. $9.2y - 4.3 = 50.9$

95. $0.05z + 0.2 = 0.15z - 10.5$

96. $0.25(60) + 0.10x = 0.15(60 + x)$

97. $0.5(3q + 87) = 1.5q + 43.5$

98. $0.4(y + 10) + 0.6y = 2$

99. $21.1w + 4.6 = 10.9w + 35.2$

100. $0.125x = 0.025(5x + 1)$

Answers

1. $x = 1$
2. $x = -13$
3. $x = 11$
4. $x = -27$
5. $x = 2$
6. $x = \frac{11}{2}$
7. $x = -2$
8. $x = -3$
9. $x = 12$
10. $x = 4$
11. $x = 6$
12. $x = -3$
13. $x = -\frac{21}{5}$
14. $x = -5$
15. $x = -19$
16. $x = -1$
17. $x = 0$
18. $x = \frac{1}{5}$
19. $x = 3$
20. $x = \frac{10}{7}$
21. $x = 0$
22. $x = -1$
23. No Solution
24. $x = -\frac{23}{5}$
25. All real numbers

26. $x = -\frac{4}{3}$
27. No Solution
28. All real numbers
29. $x = \frac{7}{3}$
30. $x = -\frac{20}{7}$
31. $x = 12$
32. $x = -5$
33. $c = 2$
34. $z = 0$
35. $a = 5$
36. $n = 30$
37. $w = 5$
38. $x = \frac{1}{4}$
39. $x = 3$
40. $a = 30$
41. $z = 30$
42. $c = -1$
43. $x = -1$
44. $x = 4$
45. $x = 16$
46. $x = -\frac{1}{5}$
47. $x = \frac{5}{8}$
48. $y = -1$
49. $x = -14$
50. $x = -48$

51. $x = 12$
52. $x = 8$
53. $x = -7$
54. $x = -56$
55. $x = 14$
56. $x = -8$
57. $x = -26$
58. $x = 5$
59. $x = -8$
60. $x = 15$
61. $x = 45$
62. $x = 9$
63. $x = 25$
64. $x = 75$
65. $x = 12$
66. $x = 144$
67. $x = 12$
68. $x = 25$
69. $x = 81$
70. $x = -9$
71. $x = -3$
72. $y = 16$
73. $x = 4$
74. $x = -\frac{1}{6}$
75. $x = \frac{1}{2}$
76. $x = 15$

77. $x = 63$
78. $x = 27$
79. $x = 72$
80. $y = 18$
81. $x = 2.5$
82. $x = 12$
83. $x = -10$
84. $x = 3$
85. $x = \frac{9}{4}$
86. $x = -\frac{13}{8}$
87. $x = 4$
88. $x = 6$
89. $x \approx -2.3$
90. All real numbers
91. No Solution
92. $x = -5$
93. $x = 6$
94. $y = 6$
95. $z = 107$
96. $x = 120$
97. All real numbers
98. $y = -2$
99. $w = 3$
100. No Solution