**Factor each polynomial completely. Write PRIME if it cannot be factored.**

1) $6a^2x^2 + 15a^2x$

2) $2x^2 + 3xy - 10x - 15y$

3) $(x-3)^2 - 4$

4) $5x^2 + 15x + 10$

5) $3x^2 + 6x + 15$

6) $16a^4 - 1$

7) $16x^2 - 8x + 1$

8) $4x^2 + 3x + 6$

9) $6x^2 + 11x - 10$

10) $3a^2 + 21b + ab + 7b$

11) $8x^2 - 2x - 15$

12) $2x^2 - 11x - 15$

13) $4x^2 + 9$

**14) $8x^3 - 27$

15) $10k^2 - 4k + 15hk - 6h$

16) $2x(x+4) - 3(x+ 4)$

17) $18x^2y - 24xy + 8y$

18) $2x^2y + 16y$

19) $9x^2 - 4y^2$

20) $4x^2 + 20x + 25$

21) $3x^2 + 13x + 14$

22) $12x^2 - 75$

# Answers

1) $3a^2x(2x + 5)$ 2) $(x - 5)(2x + 3y)$ 3) $(x - 1)(x - 5)$ 4) $5(x + 2)(x + 1)$

5) $3(x^2 + 2x + 5)$ 6) $(4a^2 + 1)(2a + 1)(2a - 1)$ 7) $(4x - 1)^2$ 8) PRIME 9) $(2x+5)(3x-2)$

10) $(3a + b)(a + 7)$ 11) $(2x-3)(4x+5)$ 12) $(x-3)(2x - 5)$ 13) PRIME

14) $(2x - 3)(4x^2 + 6x + 9)$ 15) $(2k+3h)(5k-2)$ 16) $(x + 4)(2x - 3)$ 17) $2y(3x - 2)^2$

18) $2y(x + 2)(x^2 - 2x + 4)$ 19) $(3x+2y)(3x-2y)$ 20) $(2x + 5)^2$ 21) $(x+2)(3x+7)$ 22) $3(2x+5)(2x-5)$

1) Factor: $2x^3 - 2x^2 + 3x - 3$

2) Solve by factoring: $2x^3 + 9x^2 = 5x$

3) Find the vertex of $y = 3(x-2)^2 + 7$

4) Find the discriminant and the number of solutions:

$$2x^2 - 4x - 5 = 0$$

5) Solve: $x^2 + 49 = 0$

6) Solve: $9x^2 = 49$

7) Write an equation of the line parallel to $y = \frac{3}{4}x + 7$ that goes through the point (2, 1).

8) Use the quadratic formula to solve:

$$5x^2 - 2x = -1$$

## Answers

1)$(2x^2+3)(x-1)$ 2) 2 3) (2, 7) 4) Discrim. = 56, 2 solutions 5) $\pm 7i$

6) $\pm\frac{7}{3}$ 7) $y = \frac{3}{4}x - \frac{1}{2}$ 8) $\frac{1 \pm 2i}{5}$

Find a cubic model for each function.

Then use your model to estimate the value of $y$ when $x = 7$.

1.

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $y$ | 25 | 21 | 20 | 23 | 19 | 17 |

2.

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $y$ | 3.1 | 4.2 | 4.3 | 4.4 | 5.1 | 6.7 |

Write each polynomial in standard form. Then classify it by degree and by number of terms

3. $4x + x + 2$

4. $-3 + 3x - 3x$

5. $6x^4 - 1$

6. $1 - 2s + 5s^4$

7. $5m^2 - 3m^2$

8. $x^2 + 3x - 4x^3$

9. $-1 + 2x^2$

10. $5m^2 - 3m^3$

11. $5x - 7x^2$

12. $2 + 3x^3 - 2$

13. $6 - 2x^3 - 4 + x^3$

14. $6x - 7x$

15. $a^3(a^2 + a + 1)$

16. $x(x + 5) - 5(x + 5)$

17. $p(p - 5) + 6$

18. $(3c^2)^2$

19. $-(3 - b)$

20. $6(2x - 1)$

21. $\frac{2}{3} + s^2$

22. $\frac{2x^4 + 4x - 5}{4}$

23. $\frac{3 - z^5}{3}$

24. The lengths of the sides of a triangle are $x + 4$ units, $x$ units, and $x + 1$ units. Express the perimeter of the triangle as a polynomial in standard form.

25. Find a cubic function to model the data below. (Hint: Use the number of years past 1940 for $x$.) Then use the function to estimate the average monthly Social Security Benefit for a retired worker in 2010.

Average Monthly Social Security Benefits, 1940–2003

| Year | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 | 2000 | 2003 |
|---|---|---|---|---|---|---|---|---|
| Amount (in dollars) | 22.71 | 29.03 | 81.73 | 123.82 | 321.10 | 550.50 | 844.60 | 922.10 |

Source: *www.infoplease.com*

26. Find a cubic function to model the data below. (Hint: Use $x$ to represent the gestation period.) Then use the function to estimate the longevity of an animal with a gestation period of 151 days.

Gestation and Longevity of Certain Animals

| Animal | Rat | Squirrel | Pig | Cow | Elephant |
|---|---|---|---|---|---|
| Gestation (in days) | 21 | 44 | 115 | 280 | 624 |
| Longevity (in years) | 3 | 9 | 10 | 12 | 40 |

Find the relative maximum, relative minimum, and zeros of each function. Then state the intervals on which the function is increasing or decreasing. Then state domain and range.

23. $f(x) = x^3 - 7x^2 + 10x$

24. $f(x) = x^3 - x^2 - 9x + 9$

For each function, determine the zeros. State the multiplicity of any multiple zeros.

1. $y = (x - 5)^3$
2. $y = x(x - 8)^2$
3. $y = (x - 2)(x + 7)^3$

4. $f(x) = x^4 - 8x^3 + 16x^2$
5. $f(x) = 9x^3 - 81x$
6. $y = (2x + 5)(x - 3)^2$

Write each function in standard form.

7. $y = (x - 5)(x + 5)(2x - 1)$
8. $y = (2x + 1)(x - 3)(5 - x)$

Write each expression as a polynomial in standard form.

14. $x(x - 1)^2$
15. $(x + 3)^2(x + 1)$
16. $(x + 4)(2x - 5)(x + 5)^2$

9. A rectangular box is 24 in. long, 12 in. wide, and 18 in. high. If each dimension is increased by $x$ in., write a polynomial function in standard form modeling the volume $V$ of the box.

Write a polynomial function in standard form with the given zeros.

10. $-1, 3, 4$ 11. $1, 1, 2$ 12. $-3, 0, 0, 5$ 13. $-2$ multiplicity 3

Write each function in factored form. Check by multiplication.

17. $y = 2x^3 + 10x^2 + 12x$ 18. $y = x^4 - x^3 - 6x^2$ 19. $y = -3x^3 + 18x^2 - 27x$

25. $x^3 - 6x^2 - 16x$ 26. $x^3 + 7x^2 + 12x$ 27. $x^3 - 8x^2 + 15x$

28. A rectangular box has a square base. The combined length of a side of the square base, and the height is 20 in. Let $x$ be the length of a side of the base of the box.

    a. Write a polynomial function in factored form modeling the volume $V$ of the box.

    b. What is the maximum possible volume of the box?

Divide using long division. Check your answers.

19. $(x^2 - 13x - 48) \div (x + 3)$

20. $(2x^2 + x - 7) \div (x - 5)$

21. $(x^3 + 5x^2 - 3x - 1) \div (x - 1)$

22. $(3x^3 - x^2 - 7x + 6) \div (x + 2)$

Divide using synthetic division.

5. $(x^3 - 8x^2 + 17x - 10) \div (x - 5)$

6. $(x^3 + 5x^2 - x - 9) \div (x + 2)$

7. $(-2x^3 + 15x^2 - 22x - 15) \div (x - 3)$

8. $(x^3 + 7x^2 + 15x + 9) \div (x + 1)$

9. $(x^3 + 2x^2 + 5x + 12) \div (x + 3)$

10. $(x^3 - 5x^2 - 7x + 25) \div (x - 5)$

11. $(x^4 - x^3 + x^2 - x + 1) \div (x - 1)$

12. $\left(x^4 + \frac{5}{3}x^3 - \frac{2}{3}x^2 + 6x - 2\right) \div \left(x - \frac{1}{3}\right)$

13. $(x^4 - 5x^3 + 5x^2 + 7x - 12) \div (x - 4)$

14. $(2x^4 + 23x^3 + 60x^2 - 125x - 500) \div (x + 4)$

Divide using an appropriate method.

25. $(6x^3 + 2x^2 - 11x + 12) \div (3x + 4)$

26. $(x^4 + 2x^3 + x - 3) \div (x - 1)$

27. $(2x^4 + 3x^3 - 4x^2 + x + 1) \div (2x - 1)$

28. $(x^5 - 1) \div (x - 1)$

Divide using an appropriate method.

29. $(x^4 - 3x^2 - 10) \div (x - 2)$

30. $(3x^3 - 2x^2 + 2x + 1) \div \left(x + \frac{1}{3}\right)$

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

1. $x + 4$
2. $x - 3$
3. $x + 6$
4. $x + 2$

Use synthetic division and the Remainder Theorem to find $P(a)$.

15. $P(x) = 3x^3 - 4x^2 - 5x + 1$; $a = 2$

16. $P(x) = x^3 + 7x^2 + 12x - 3$; $a = -5$

17. $P(x) = x^3 + 6x^2 + 10x + 3$; $a = -3$

18. $P(x) = 2x^4 - 9x^3 + 7x^2 - 5x + 11$; $a = 4$

Use synthetic division and the given factor to completely factor each polynomial function.

23. $y = x^3 + 3x^2 - 13x - 15$; $(x + 5)$

24. $y = x^3 - 3x^2 - 10x + 24$; $(x - 2)$

31. A box is to be mailed. The volume in cubic inches of the box can be expressed as the product of its three dimensions: $V(x) = x^3 - 16x^2 + 79x - 120$. The length is $x - 8$. Find linear expressions for the other dimensions. Assume that the width is greater than the height.

Use long division:

1) $(3x^3 + x^2 - 4x + 2) \div (x - 1)$

Ans._____

2) $(2x^3 + 3x^2 - 4x - 7) \div (x^2 - 2)$

Ans._____

3) $(12x^4 - 5x^2 - 3) \div (x - 2)$

Ans._____

Use synthetic division:

4) $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

Use synthetic division:

5) $(x^3 - 2x^2 - 19) \div (x - 3)$

Ans._____

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Use the remainder theorem to evaluate the function.

6) $f(x) = 3x^3 - 7x^2 + 4x - 2$ when $x = -2$

**f(  ) =_____**

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7) $f(x) = 2x^3 + 6x^2 - 8$ when $x = 1$

f(  ) =_____

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8) Given $f(x) = x^3 + 3x^2 - 4$ and one factor is $(x + 2)$. Find:

**a) remaining factors**

**b) all of the zeros/roots**

## Answers

1) $2x^2 + 3x - 1 + 1/(x-1)$ 2) $2x + 3 + -1/(x^2 - 2)$ 3) $12x^3 + 24x^2 + 43x + 86 + 169/(x-2)$
4) $3x^3 - 5x + 2$ 5) $x^2 + x + 3 + -10/(x-3)$ 6) $-62$ 7) 0 8a) $(x+2)(x-1)$ 8b) -2,-2,1

**Factor each expression.**

25. $x^3 - 125$

26. $x^4 - 8x^2 + 15$

27. $x^4 + x^2 - 2$

28. $x^3 + 1$

29. $x^4 - 2x^2 - 24$

30. $x^4 + 10x^2 + 9$

31. $x^3 + 27$

32. $x^4 + 7x^2 - 18$

**Factor the expression on the left side of each equation. Then solve the equation.**

1. $8x^3 - 27 = 0$

2. $x^3 + 64 = 0$

3. $2x^3 + 54 = 0$

4. $2x^3 - 250 = 0$

**Factor the expression on the left side of each equation. Then solve the equation.**

5. $4x^3 - 32 = 0$

6. $27x^3 + 1 = 0$

7. $64x^3 - 1 = 0$

8. $x^3 - 27 = 0$

9. $x^4 - 5x^2 + 4 = 0$

10. $x^4 - 12x^2 + 11 = 0$

11. $x^4 - 10x^2 + 16 = 0$

12. $x^4 - 8x^2 + 16 = 0$

13. $x^4 - 9x^2 + 14 = 0$

14. $x^4 + 13x^2 + 36 = 0$

15. $x^4 - 10x^2 + 9 = 0$

16. $x^4 + 3x^2 - 4 = 0$

**Solve each equation.**

33. $x^4 - x = 0$

34. $3x^4 + 18 = 21x^2$

35. $2x^4 - 26x^2 - 28 = 0$

36. $5x^4 + 50x^2 + 80 = 0$

**Solve each equation.**

37. $x^4 - 81 = 0$

38. $x^4 = 25$

39. $x^5 = x^3 + 12x$

40. $x^4 + 12x^2 = 8x^3$

17. **Over 3 yr, Lucia saved \$550, \$600, and \$650 from baby-sitting jobs. The polynomial $550x^3 + 600x^2 + 650x$ represents her savings, with interest, after 3 yr. The annual interest rate equals $x - 1$. Find the interest needed so she will have \$2000 after 3 yr.**

**Solve each equation by graphing. Where necessary, round to the nearest hundredth. (Use 2nd calc zero or 2nd calc intersect)**

18. $2x^4 = 9x^2 - 4$

19. $x^2 - 16x = -1$

20. $6x^3 + 10x^2 + 5x = 0$

21. $36x^3 + 6x^2 = 9x$

22. $15x^4 = 11x^3 + 14x^2$

23. $x^4 = 81x^2$

24. **The product of three consecutives integers $n - 1$, $n$, and $n + 1$ is $-336$. Write and solve an equation to find the numbers.**

Perform the indicated operations. Put your answers in standard form. Then classify the polynomial by degree and number of terms

**1.** $\left(3n^2+5n-6\right)+\left(-n^2-3n+3\right)$

**2.** $\left(3x^2-4x-2\right)-\left(-x^2-4x+7\right)$

**3.** $3x\left(x^2-2x+4\right)$

**4.** $(2x-1)(x-5)$

**5.** $(2x-1)\left(x^2-x+3\right)$

**6.** $(x-5)^3$

Put each polynomial in standard form, state its degree, leading term and whether it is a monomial, binomial, trinomial or polynomial (more than 4 terms).

| | Standard Form | degree | Leading term | classify |
|---|---|---|---|---|
| **7.** $5x+7x^3-2x$ | | | | |
| **8.** $4x^2+10+2x-2x^2$ | | | | |
| **9.** $8y^5-5y^6+7y^5-15y^5$ | | | | |

Factor completely. If the expression is prime, say so.

**9.** $18y^3 + 24y^2 + 8y$

**10.** $6c^3 - 16c^2 + 10c$

**11.** $5u^2 - 6u - 2$

**12.** $y^4 + 8y^2 - 20$

**13.** $p^3 - 2p^2 + 4p - 8$

**14.** $8x^3 + 27$

**15.** $a^2bc - 4bc + a^2b - 4b$
**(Hint: GCF, then factor by grouping)**

**16.** $-4n^4 + 40n^3 - 100n^2$
**(Hint: Use a negative GCF)**

**17.** $125x^3 - 64$

**18.** $x^5 + 14x^3 + 13x$

Solve each equation for all zeros in the complex plane.

**19.** $y^2 - 3y + 2 = 0$

**20.** $4x^3 - 12x^2 + 8x = 0$

**21.** $k^2 + 9 = 10k$

**22.** $y^4 - 10y^2 + 9 = 0$

**23.** $4x^4 - 2x^2 - 4 = 2$

**24.** $64x^3 + 8 = 0$

A polynomial equation with rational coefficients has the given roots. Find two additional roots.

1. $2 + 3i$ and $\sqrt{7}$

2. $3 - \sqrt{2}$ and $1 + \sqrt{3}$

3. $-4i$ and $6 - i$

4. $5 - \sqrt{6}$ and $-2 + \sqrt{10}$

Find a fourth-degree polynomial equation with integer coefficients that has the given numbers as roots.

5. $2i$ and $4 - i$

6. $\sqrt{2}$ and $2 - \sqrt{3}$

7. $3i$ and $\sqrt{6}$

8. $2 + i$ and $1 - \sqrt{5}$

Find the roots of each polynomial equation.

9. $x^3 - 5x^2 + 2x + 8 = 0$

10. $x^3 + x^2 - 17x + 15 = 0$

11. $2x^3 + 13x^2 + 17x - 12 = 0$

12. $x^3 - x^2 - 34x - 56 = 0$

13. $x^3 - 18x + 27 = 0$

14. $x^4 - 5x^2 + 4 = 0$

15. $x^3 - 6x^2 + 13x - 10 = 0$

16. $x^3 - 5x^2 + 4x + 10 = 0$

17. $x^3 - 5x^2 + 17x - 13 = 0$

18. $x^3 + x + 10 = 0$

19. $x^3 - 5x^2 - x + 5 = 0$

20. $x^3 - 12x + 16 = 0$

21. $x^3 - 2x^2 - 5x + 6 = 0$

22. $x^3 - 8x^2 - 200 = 0$

23. $x^3 + x^2 - 5x + 3 = 0$

24. $4x^3 - 12x^2 - x + 3 = 0$

25. $x^3 + x^2 - 7x + 2 = 0$

26. $12x^3 + 31x^2 - 17x - 6 = 0$

Use the Rational Root Theorem to list all possible rational roots for each polynomial equation. Then find any actual rational roots.

27. $x^3 + 5x^2 - 2x - 15 = 0$

28. $36x^3 + 144x^2 - x - 4 = 0$

29. $2x^3 + 5x^2 + 4x + 1 = 0$

30. $12x^4 + 14x^3 - 5x^2 - 14x - 4 = 0$

31. $5x^3 - 11x^2 + 7x - 1 = 0$

32. $x^3 + 81x^2 - 49x - 49 = 0$

Find a third-degree polynomial equation with rational coefficients that has the given numbers as roots. (Hint: conjugate pairs)

33. $3, 2 - i$

34. $5, 2i$

35. $-1, 3 + i$

36. $-7, i$

37. $-4, 4i$

38. $6, 3 - 2i$

**Find all the zeros of each function.**

1. $y = 5x^3 - 5x$
2. $f(x) = x^3 - 16x$
3. $g(x) = 12x^3 - 2x^2 - 2x$
4. $y = 6x^3 + x^2 - x$
5. $f(x) = 5x^3 + 6x^2 + x$
6. $y = -4x^3 + 100x$

**For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.**

7. $2x^2 + 5x + 3 = 0$
8. $3x^2 + 11x - 10 = 0$
9. $2x^4 - 18x^2 + 5 = 0$
10. $4x^3 - 12x + 9 = 0$
11. $6x^5 - 28x + 15 = 0$
12. $x^3 - x^2 - 2x + 7 = 0$