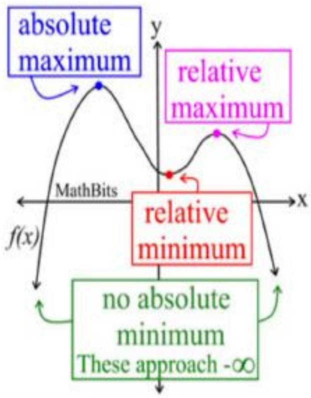
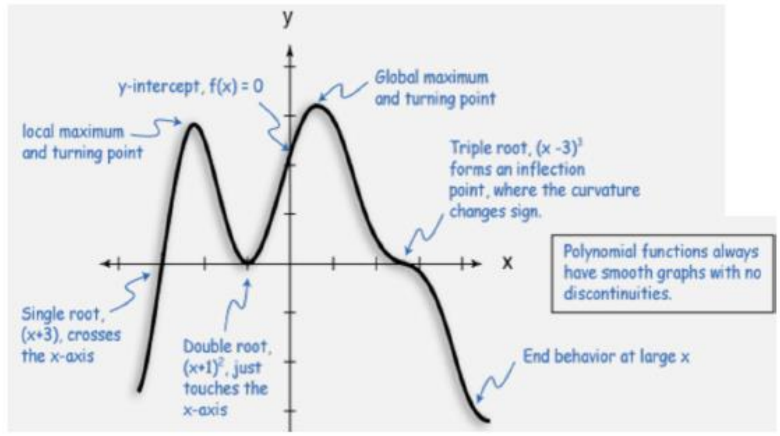




There are special names we give to polynomials according to their degree and number of terms.

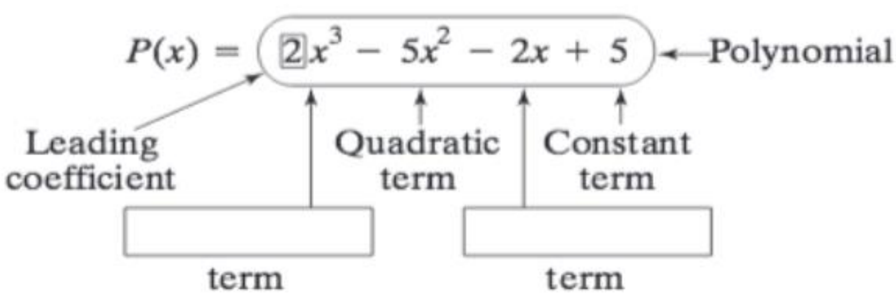
Degree	Name of Degree	Example		Number of Terms	Name	Example
0	Constant			1	Monomial	
1	Linear			2	Binomial	
2	Quadratic			3	Trinomial	
3	Cubic			4	Polynomial of 4 terms	
4	Quartic			n	Polynomial of n terms	
5	Quintic					
n	nth degree					

Vocabulary and Key Concepts

Polynomial Function

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ where n is a nonnegative integer and the coefficients $a_n, \ldots, a_0$ are ______ numbers.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	6	1	monomial
	linear	$x + 6$	2	binomial
2		$3x^2$	1	monomial
3	cubic	$2x^3 - 5x^2 - 2x$		trinomial
4	quartic	$x^4 + 3x^2$	2	
	quintic	$-2x^5 + 3x^2 - x + 4$	4	polynomial of 4 terms



1. Write each polynomial in standard form. Then classify each polynomial by its degree and number of terms. Finally, name the leading coefficient of each polynomial.

a. $9 + x^2$

b. $x^3 - 2x^2 - 3x^4$

More Examples:

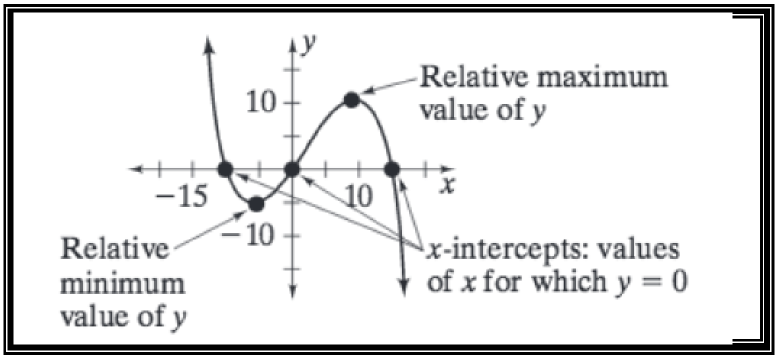
c. $-7x + 5x^4$

d. $x^2 - 4x + 3x^3 + 2x$

e. $4x - 6x + 5$

f. $6 - 3x^5$

Identify the characteristics of a polynomial function, such as the intervals of increase/decrease, intercepts, domain/range, relative minimum/maximum, and end behavior.



Relative Maximum – the greatest y-value among the nearby points on the graph.
Relative Minimum – the smallest y-value among the nearby points on the graph.

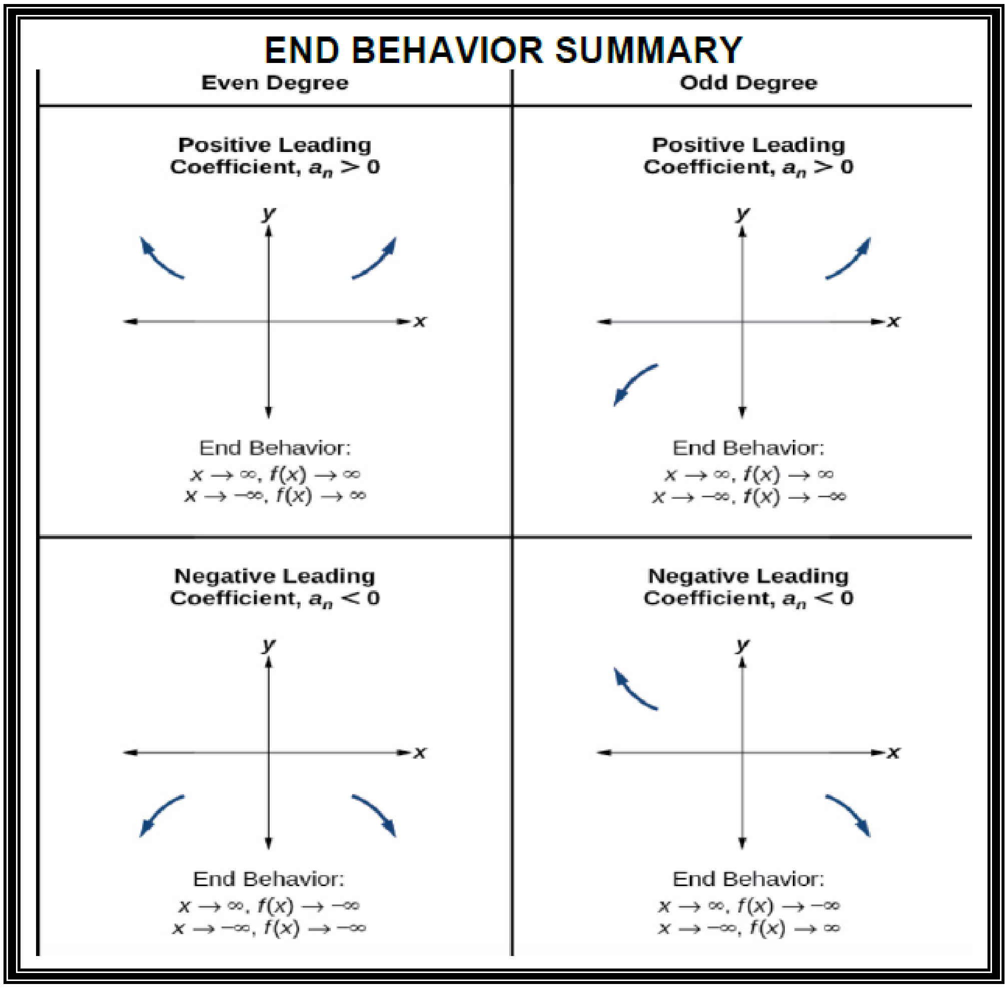
Multiple Zero – a zero of a linear factor that is repeated in the factored form of the polynomial

Multiplicity of a Zero – the number of times the related linear factor is repeated in the factored form of a polynomial.
-Impacts the behavior of the graph around the x-intercept (bounce, cross)

Domain: all possible x or input values
Range: all possible y or output values

Intervals of Increasing– the x values for which the y value are increasing
Intervals of Decreasing– the x values for which the y value are decreasing

	Quick Sketch of Function	Is the function always increasing, always decreasing, some of both, or neither?	What is the *largest* number of x-intercepts that the function can have?	What is the *smallest* number of x-intercepts that the function can have?	Domain
Constant Function					
1st Degree $f(x) = x$					
2nd Degree $f(x) = x^2$					
3rd Degree $f(x) = x^3$					
4th Degree $f(x) = x^4$					


END BEHAVIOR SUMMARY
Even Degree
Odd Degree
Positive Leading Coefficient, $a_n > 0$
End Behavior:
$x \to \infty$, $f(x) \to \infty$
$x \to -\infty$, $f(x) \to \infty$
Positive Leading Coefficient, $a_n > 0$
End Behavior:
$x \to \infty$, $f(x) \to \infty$
$x \to -\infty$, $f(x) \to -\infty$
Negative Leading Coefficient, $a_n < 0$
End Behavior:
$x \to \infty$, $f(x) \to -\infty$
$x \to -\infty$, $f(x) \to -\infty$
Negative Leading Coefficient, $a_n < 0$
End Behavior:
$x \to \infty$, $f(x) \to -\infty$
$x \to -\infty$, $f(x) \to \infty$

Ex: 1. Describe the end behavior of the graph of each polynomial function by completing the statements and sketching arrows. Do this without looking at the graph.

a) $f(x) = -x^6 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

b) $f(x) = 2x^3 + 2x^2 - 5x - 10$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

c) $f(x) = -2x^5 + x^2 - 1$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

d) $f(x) = x^4 - 5x + 10$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

<u>Quick Check:</u> Describe the end behavior of the graph of each polynomial function by completing the statements and sketching arrows. Do this without looking at the graph.

1) $f(x) = -5x^6 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

2) $f(x) = 2x^5 + 2x^3 – 5x -6$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

3) $f(x) = 3x^4 + 4x^2 + 2$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

4) $f(x) = -2x^3 + 2x^2 – 5x -6$

as x-> -∞ f(x) ->

as x-> +∞ f(x) ->

Summary of Minimums and Maximums

A relative minimum or maximum is a point that is the min. or max. relative to other nearby function values. (Note: Parabolas had an absolute min or max)

- Approximate the min or max *(First adjust your window as needed for your graph)*
1) Press 2nd TRACE, then press MIN or MAX (depending on the shape of your function).

2) Move your cursor just to the "left" of the relative min or relative max. Press ENTER.

3) Move your cursor just to the "right" of the relative min or relative max. Press ENTER.

4) The screen will show "Guess". Press ENTER again.

5) The bottom of the screen will say X=____ Y =_______
The y value is the relative min or relative max. The x value is where the min or max is occurring.
The relative min or relative max in this example is _________ at _______.

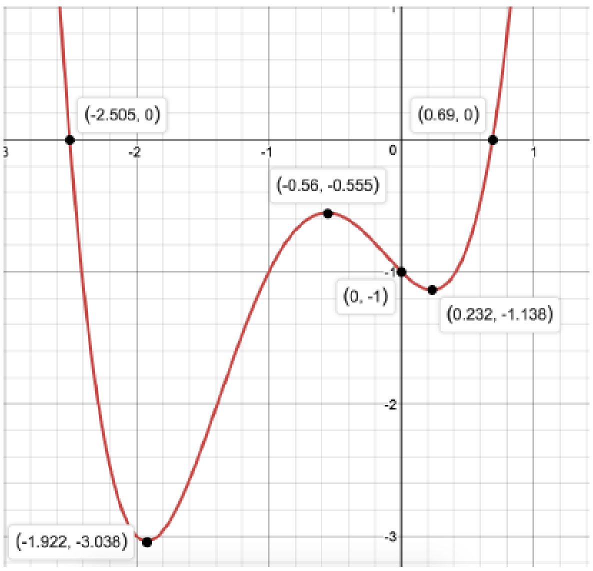
Ex 2: Graph the equation $y = 3x^3 - 5x + 5$ in your calculator. Then determine the coordinates of all relative minimums and maximums (rounded to 3 decimal places).

Quick Check Determine the coordinates of all relative minimums and maximums (rounded to 3 decimal places).

a. $y = 0.5x^4 - 3x^2 + 3$

b. $f(x) = -x^3 + 6x^2 - x - 1$

Ex 3: Determine the intervals of increase and decrease, the intercepts, the domain and range, and the coordinates of all relative minimums and maximums. Round all answers to three decimal places.



Intervals of increase: ____________________

Intervals of decrease: ________________________

y-Intercept: ____________________

x-Intercepts: ____________________________

Domain: ____________

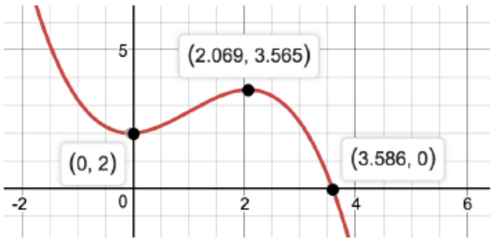
Range: ________________

Relative Minimum(s): _____________________

Relative Maximum(s): ____________________

Quick Check Determine the intervals of increase and decrease, the intercepts, the domain and range, and the coordinates of all relative minimums and maximums. Round all answers to three decimal places.

a)



Intervals of increase: ___________________________

Intervals of decrease: ___________________________

y-Intercept: ___________________

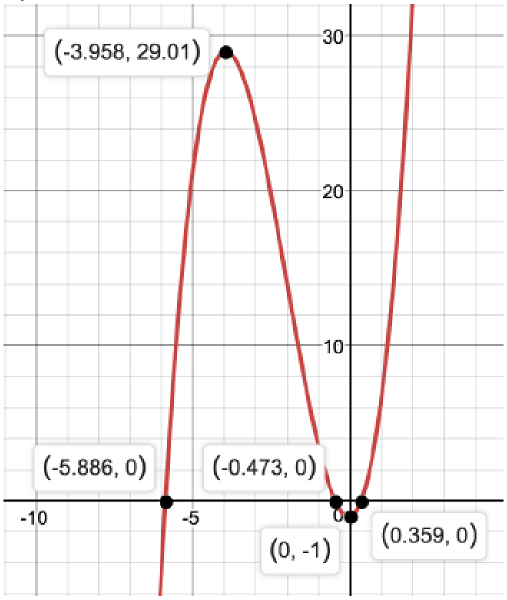
x-Intercepts: ___________________________

Domain: ____________ Range: ________________

Relative Minimum(s): _________________________

Relative Maximum(s): _________________________

b)



Intervals of increase: ____________________

Intervals of decrease: ____________________

y-Intercept: ______________

x-Intercepts: ____________________

Domain: _________ Range: ____________

Relative Minimum(s): ____________________

Relative Maximum(s): ____________________

More Practice.
Further analyze the graphs of the functions.

a. $y = 0.5x^4 - 3x^2 + 3$

Intervals of increase: ____________________

Intervals of decrease: ____________________

y-Intercept: ______________ x-Intercepts: __________________________

Domain: _________ Range: ____________

Relative Minimum(s): ____________________

Relative Maximum(s): ____________________

---

b. $f(x) = -x^3 + 6x^2 - x - 1$

Intervals of increase: ____________________

Intervals of decrease: ____________________

y-Intercept: ______________ x-Intercepts: ____________________

Domain: _________ Range: ____________

Relative Minimum(s): ____________________

Relative Maximum(s): ____________________

I can use polynomial functions to model real life situations and make predictions

Comparing Models Use a graphing calculator to find the best regression equation for the following data. Compare linear, quadratic and cubic regressions.

x	**0**	**5**	**10**	**15**
y	**10.1**	**2.8**	**8.1**	**16.0**

Recall finding a regression equation using STAT: (turn diagonstics on so r values are calculated)
- Enter your data (STAT, Edit…)
- Turn on Stat Plot 1 (2^{nd}, STAT PLOT)
- Graph the data (ZOOM, 9)
- STAT, right, then choose your desired regression.
- Your command should look like ____Reg, Y_1 (VARS, right, ENTER, ENTER)

Which regression model is best? Why?

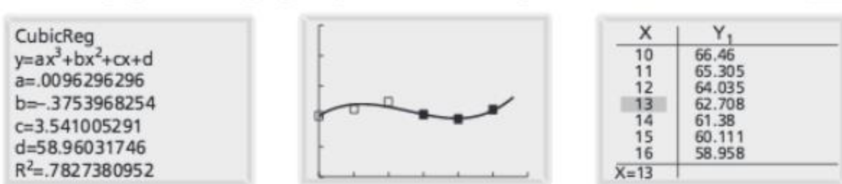
Predict the value of y when x = 12.

Example

3 Using Cubic Functions The table shows data on the number of employees that a small company had from 1975 to 2000. Find a cubic function to model the data. Use it to estimate the number of employees in 1988.

Year	Number of Employees
1975	60
1980	65
1985	70
1990	60
1995	55
2000	64

Enter the data. Let 0 represent 1975. To find a cubic model, use the CubicReg option of a graphing calculator. Graph the model.



The function $f(x) = 0.00963x^3 - 0.3754x^{\square} + 3.541x + 58.96$ is an approximate model for the ____ function.

To estimate the number of employees for 1988, you can use the Table option of a graphing calculator to find that $f(13) \approx 62.71$. According to the model, there were about ____ in 1988.

QUICK CHECK:

1. Use the cubic model in Example 3 to estimate the number of employees in 1999.

2. The table below shows world gold production for several years.

World Gold Production

Year	1975	1980	1985	1990	1995	2000
Production (millions of troy ounces)	38.5	39.2	49.3	70.2	71.8	82.6

SOURCES: *The World Almanac* and *World Gold*

a. Enter the data in your calculator. Let 0 represent 1975. Graph the data and sketch it here:

b. Find a quartic function to model the data: ____

c. Use your model to predict the gold production in 2018. _________________________

**NOTE: Some real life situations will not require a calculator to write an equation.*

3. Travel: Several popular models of carry-on luggage have a length of 10 inches greater than their depth. To comply with airline regulations, the sum of the length, width and depth may not exceed 40 inches.

a. Assume the sum of the length, width, and depth is 40 in. Write a function for the volume of the luggage.

b. What is the maximum possible volume of the luggage? What dimensions correspond to this volume?

Length: ___________ Width: _____________ Depth: ______________

Factors and Zeros

After this lesson and practice, I will be able to …
LT 4. write standard form polynomial equations in factored form and vice versa.
LT 5 find the zeros (or x-intercepts or solutions) of a polynomial in factored form and identify the multiplicity of each zero.
LT 6 write a polynomial function from its real roots.

LT 4. I can write standard form polynomial equations in factored form and vice versa.

Factor Theorem

The expression *x-a* is a linear factor of a polynomial if and only if the value of *a* is a ________________ of the related polynomial function.

Equivalent Statements about Polynomials

① -4 is a __________ of $x^2 + 3x - 4 = 0$.

② -4 is an __________ of the graph of $y = x^2 + 3x - 4$.

③ -4 is a __________ of $y = x^2 + 3x - 4$.

④ $x + 4$ is a __________ of $x^2 + 3x - 4$.

Writing a Polynomial in Standard Form.

Recall: **Standard Form of a Polynomial:** The term with the highest degrees first and place in the other terms in descending order.

1) Write (x - 1)(x + 3)(x + 4) as a polynomial in standard form.

QUICK CHECK:

2. Write each expression in standard form.

a. $(x-3)(x+2)(x-4)$

b. $(x+5)(x-1)(x+2)$

3. Write the expression $(x + 1)(x - 1)(x + 2)$ as a polynomial in standard form.

Writing a Polynomial in Factored Form.

1) Write $3x^3 - 18x^2 + 24x$ as a polynomial in factored form. Check by multiplication or on the calculator.

QUICK CHECK:

2) Write each polynomial in factored form.

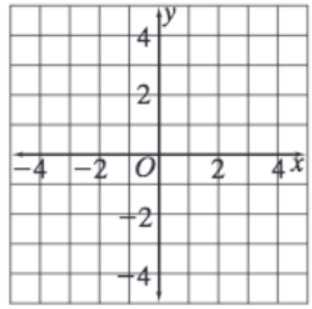
a) $3x^3 - 3x^2 - 36x$

b) $2x^3 + 10x^2 + 12x$

c) $6x^3 - 15x^2 - 36x$

I can find the zeros (or x-intercepts or solutions) of a polynomial in factored form and identify the multiplicity of each zero

Finding the Zeros of a polynomial Function.



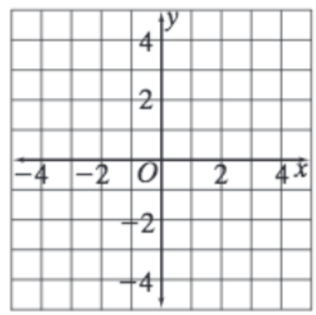
3) Find the zeros of the polynomial function

f(x) = (x + 1)(x - 1)(x + 3)

The Zeros: ___________, ___________ and ___________

Sketch the Function. Plot the x and y intercepts.
Hint: y intercept is when x = 0. (0,?)

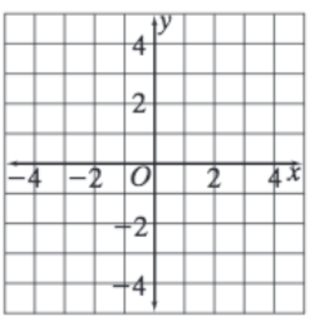
QUICK CHECK:
Find the zeros of each function. Then sketch a graph of the function, showing x and y intercepts.



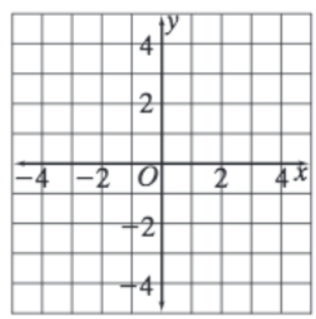
a. Find the zeros of the polynomial function
y = (x + 2)(x - 5)(x - 3)

The Zeros of the Function: ______, ______ and ______
Sketch the Function. Plot the x and y intercepts.

b. $y=(x-1)(x+2)(x+1)$



c. $y=(x-4)(x-1)(x+2)$



Finding the Multiplicity of a Zero

Recall:

Multiple Zero – a zero of a linear factor that is repeated in the factored form of the polynomial

Multiplicity of a Zero – the number of times the related linear factor is repeated in the factored form of a polynomial.
-Impacts the behavior of the graph around the x-intercept (bounce or cross)

Find the zeros of the function $y = (x - 1)(x + 2)(x + 2)$.

Notice that while the function has three total zeros, only __________ of then were __________ (______ is listed as a zero twice). When a zero (and thus its linear factor) is repeated, it is called a _________________ zero. In this example, -2 is said to have multiplicity _________ since it occurs as a zero ____________.

In general, the ______________________ of a zero is equal to the number of times the zero occurs.

QUICK CHECK:

1. Find all zeros of $f(x) = x^4 + 6x^3 + 8x^2$ and state the multiplicity of each zero.

2. Find the zeros and its multiplicity and the y-intercept for the function $f(x) = (x-5)(x-4)^3(x+1)^2$

3. Sketch: $y = 5(x-1)(x+2)^3(x+5)^3$

State it's degree and leading coefficient.
Describe it's end behaviors.

I can write a polynomial function from its real roots.

Writing a Polynomial Function From Its Zeros.

Recall: We did this with quadratic functions already.

1) Write a polynomial function in **standard form** with zeros at. 2, -3 and 0.

QUICK CHECK:

2) Write a polynomial function in standard form with zeros -4,-2,0.

3) Is there another polynomial function with zeros -4, -2, 0? Explain.

4) Write a polynomial function in standard form with zeros $\sqrt{5}, -\sqrt{5}$, -5, 1.
(Remember: Irrational zeros and complex zeros always come in conjugate pairs)

5) Write a polynomial equation in standard form with zeros at -2, 3, and 3.

6) Write a polynomial equation in standard form with zeros at 2, 1, and -3.