Examples

Example 1: Find $\sqrt[3]{(7-\sqrt{3})^3}$ exactly

Solution

Since the index is **odd**, we use $\sqrt[n]{x^n} = x$ in **this** case

Thus $\sqrt[3]{(7-\sqrt{3})^3} = 7-\sqrt{3}$

Ans $\boxed{7-\sqrt{3}}$

Example 2: Find $\sqrt[4]{(10-\sqrt{5})^4}$ exactly

Solution

Since the index is **even**, we use $\sqrt[n]{x^n} = |x|$ in **this** case

Thus $\sqrt[4]{(10-\sqrt{5})^4} = |10-\sqrt{5}|$

Now $10-\sqrt{5} \geq 0$; so we use $|x| = x$ in **this** case

Thus $|10-\sqrt{5}| = 10-\sqrt{5}$

Ans $\boxed{10-\sqrt{5}}$

Example 3: Find $\sqrt[6]{(1-\sqrt{7})^6}$ exactly

Solution

Since the index is **even**, we use $\sqrt[n]{x^n} = |x|$ in **this** case

Thus $\sqrt[6]{(1-\sqrt{7})^6} = |1-\sqrt{7}|$

Now $1-\sqrt{7} < 0$; so we use $|x| = -x$ in **this** case

Thus $|1-\sqrt{7}| = -(1-\sqrt{7})$

Ans $\boxed{-1+\sqrt{7}}$

Example 4: Find $\sqrt{(\sqrt[3]{6}-\sqrt[3]{13})^2}$ exactly

Solution

Since the index is **even**, we use $\sqrt[n]{x^n} = |x|$ in **this** case

Thus $\sqrt{(\sqrt[3]{6}-\sqrt[3]{13})^2} = |\sqrt[3]{6}-\sqrt[3]{13}|$

Now $\sqrt[3]{6}-\sqrt[3]{13} < 0$; so we use $|x| = -x$ in **this** case

Thus $|\sqrt[3]{6}-\sqrt[3]{13}| = -(\sqrt[3]{6}-\sqrt[3]{13})$

Ans $\boxed{-\sqrt[3]{6}+\sqrt[3]{13}}$