OPERATIONS OF RATIONAL EXPRESSIONS

Complete the operation or simplify.

1. $\dfrac{4x+12}{x^2-2x-15}$

$\dfrac{4(x+3)}{(x-5)(x+3)}$

$\boxed{\dfrac{4}{x-5}}$

2. $\dfrac{15x^3y^5}{2xy} \cdot \dfrac{6xy^3}{9x^4y}$

$\dfrac{30x^4y^8}{6x^5y^2}$

$\boxed{\dfrac{5y^6}{x}}$

3. $\dfrac{8x^3-125}{x^2-25} \cdot \dfrac{x+5}{2x-5}$

$\dfrac{(2x-5)(4x^2+10x+25)}{(x-5)(x+5)} \cdot \dfrac{(x+5)}{(2x-5)}$

$\boxed{\dfrac{4x^2+10x+25}{x-5}}$

4. $\left(\dfrac{x^2+6x}{x^2+8x+12}\right)\left(\dfrac{x^2+2x-8}{x^3-8}\right)$

$\dfrac{x(x+6)}{(x+6)(x+2)} \cdot \dfrac{(x+4)(x-2)}{(x-2)(x^2+2x+2)}$

$\boxed{\dfrac{x(x+4)}{(x+2)(x^2+2x+4)}}$

5. $\dfrac{\dfrac{x^2-9x-22}{x^2+5x-24}}{\dfrac{x+2}{x-3}}$

$\dfrac{x^2-9x-22}{x^2+5x-24} \div \dfrac{x+2}{x-3}$

$\dfrac{(x-11)(x+2)}{(x+8)(x-3)} \cdot \dfrac{(x-3)}{(x+2)}$

$\boxed{\dfrac{x-11}{x+8}}$

6. $\dfrac{4x}{x+3} - \dfrac{4x+1}{x+3}$

$\dfrac{4x-(4x+1)}{x+3}$

$\dfrac{4x-4x-1}{x+3}$

$\boxed{\dfrac{-1}{x+3}}$

7. $\dfrac{5x}{(x-2)(x+2)} + \dfrac{(x+3)}{(x-2)}$ LCD $= (x+2)(x-2)$

$\dfrac{5x(x-2)}{(x-2)(x+2)} + \dfrac{(x+3)(x+2)}{(x-2)(x+2)}$

$\dfrac{5x(x-2)+(x+3)(x+2)}{(x+2)(x-2)}$

$\dfrac{5x^2-10x+x^2+5x+6}{(x+2)(x-2)}$

$\boxed{\dfrac{6x^2-5x+6}{(x+2)(x-2)}}$

36

1 36
2 18
3 12
+4 −9
6 6

8. $\dfrac{5\cdot 5x}{5\cdot 3y} - \dfrac{2\cdot y}{15\cdot y}$ LCD $= 15y$

$\dfrac{25x}{15y} - \dfrac{2y}{15y}$

$\boxed{\dfrac{25x-2y}{15y}}$

LCD = $x(x+3)$

9. $\frac{2x}{x+3} + \frac{5}{x^2+3x}$

$\frac{(x)(2x)}{(x)(x+3)} + \frac{5}{x(x+3)}$

$\frac{2x^2}{x(x+3)} + \frac{5}{x(x+3)}$

$\boxed{\frac{2x^2+5}{x(x+3)}}$

LCD = $12(x-4)(x+3)$

10. $\frac{x}{x^2-x-12} \quad \frac{5}{12x-48}$

$\frac{12 \cdot x}{12(x-4)(x+3)} - \frac{5(x+3)}{12(x-4)(x+3)}$

$\frac{12x}{12(x-4)(x+3)} - \frac{5(x+3)}{12(x-4)(x+3)}$

$\frac{12x-5(x+3)}{12(x-4)(x+3)}$

$\frac{12x-5x-15}{12(x-4)(x+3)}$

$\boxed{\frac{7x-15}{12(x-4)(x+3)}}$

Factor each expression completely.

11. $64x^2 - 121$

$A = 8x$
$B = 11$

$\boxed{(8x-11)(8x+11)}$

12. $x^3 - 125$

$\boxed{(x-5)(x^2+5x+25)}$

13. $6x^2 - 7x - 5$

$6x^2+3x-10x-5$
$3x \quad -5$

$3x(2x+1)-5(2x+1)$

$\boxed{(2x+1)(3x-5)}$

30
1 30
2 15
+3 −10
5 6

14. $x^2 - 15x + 50$

$\boxed{(x-10)(x-5)}$

15. $3x^3 - 24$

3

$3(x^3-8)$

$3(x-2)(x^2+2x+4)$

16. $x^2 - 13x - 30$

$(x+2)(x-15)$

30

1 30

+2 −15

~~−3 +10~~

5 6

17. $27x^3 + 64$

$A = 3x$

$B = 4$

$(3x+4)(9x^2-12x+16)$

18. $x^2 - 1$

$(x-1)(x+1)$

19. $x^2 - 64x$

x

$x(x-64)$

20. $20x^2 + 26x - 6$

2

$2(10x^2+13x-3)$

$2(10x^2+15x-2x-3)$

$5x \quad -1$

$2(5x(2x+3)-1(2x+3))$

$2(2x+3)(5x-1)$

30

1 30

−2 +15

~~+3 +10~~

5 6

21. $24x^4 - 8x^3 + 4x$

$4x$

$4x(6x^3-2x^2+1)$

22. $4x^2 + 49$

PRIME!

23. $3x^2 + 24x + 48$

3

$3(x^2+8x+16)$

$3(x+4)(x+4)$

$3(x+4)^2$

16

1 16

2 8

+4 +4

24. $25x^2 - 60x + 36$

$(5x-6)^2$

check: $(5x-6)(5x-6)$

$25x^2-30x-30x+36$ ✓

SECTION 2: GRAPHING RATIONAL FUNCTIONS

1. How do you find a vertical asymptote of a rational function?
   Set the denominator equal to zero. Solve!

2. How do you find a horizontal asymptote when graphing if the degree is larger on the denominator?
   H is always $y = 0$ (11)

3. How do you find a horizontal asymptote when graphing if the degree of the numerator is the same as the degree of the denominator?
   y = ratio of leading coeff.

4. How do you find a removable discontinuity of a rational function?
   If a factor cancels, set that factor equal to zero and solve for x. Plug x into the function to find y. Plot on the graph!

5. How do you find the domain and range of a rational function?
   Domain: $\mathbb{R}$, $x \neq$ VA or x-coordinate of hole.
   Range: $\mathbb{R}$, $x \neq$ HA or y-coordinate of hole.

6. How do you find the degree of a polynomial in standard form? $y = ax^3 + bx + c \cdots$
   The highest exponent.

7. How do you find the degree of a polynomial in factored form? $y = (\quad)(\quad)$
   Count the x's.
   (Sum of the exponents)

Circle the asymptotes of the following function. Circle all that apply.

8. $y = \frac{3}{x-1} + 4$ $x = 1$, $y = 4$

   (x = 1) x = −1 x = 4 y = 3 y = 1 (y = 4)

9. $y = \frac{10}{x^2 - 25}$ $x^2 - 25 = 0$, $x^2 = 25$, $x = \pm 5$. Degree larger in the denominator. $y = 0$

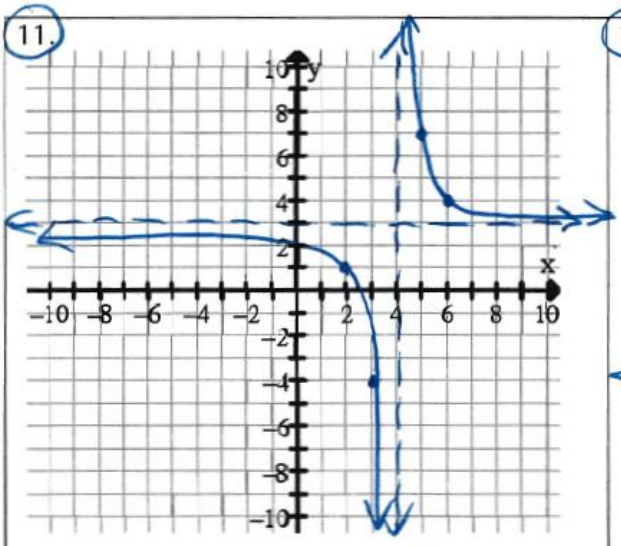
   (x = 5) x = 0 (x = −5) (y = 0) y = 1 y = 5

10. Identify the vertical asymptote, horizontal asymptote, domain and range of each equation.

	Vertical Asymptote	Horizontal Asymptote	Domain	Range
a. $y=\frac{4}{x}$	$x=0$	$y=0$	$\mathbb{R}, x\neq 0$	$\mathbb{R}, y\neq 0$
b. $y=\frac{1}{x+2}-3$	$x=-2$	$y=-3$	$\mathbb{R}, x\neq -2$	$\mathbb{R}, y\neq -3$
c. $y=\frac{3x-6}{x+2}$	$x=-2$	$y=3$	$\mathbb{R}, x\neq -2$	$\mathbb{R}, y\neq 3$
d. $f(x)=\frac{6x-1}{3x+6}$	$x=-2$	$y=2$	$\mathbb{R}, x\neq -2$	$\mathbb{R}, y\neq 2$

Identify the vertical asymptote, horizontal asymptote, domain and range of the graph.

11.



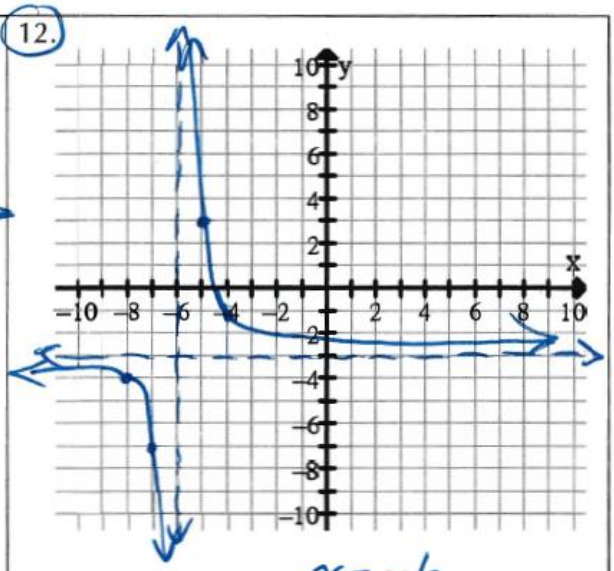
Vertical Asymptote: $x=4$

Horiztonal Asymptote: $y=3$

Domain: $\mathbb{R}, x\neq 4$

Range: $\mathbb{R}, y\neq 3$

12.



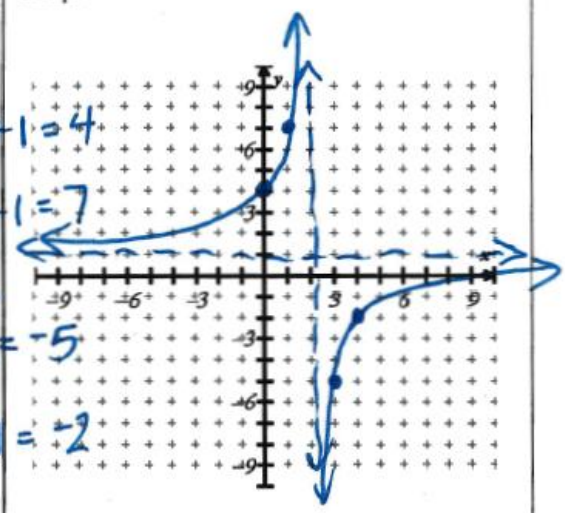
Vertical Asymptote: $x=-6$

Horiztonal Asymptote: $y=-3$

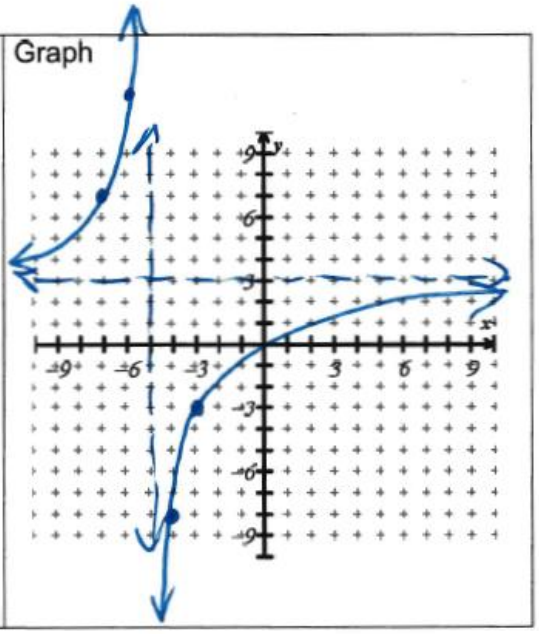
Domain: $\mathbb{R}, x\neq -6$

Range: $\mathbb{R}, y\neq -3$

13. $y = \dfrac{-6}{x-2} + 1$

VA:	Domain:	Table	Graph
$x-2=0$ $x=2$	$\mathbb{R}, x \neq 2$	x | y 0 | $\frac{-6}{-2}+1=4$ 1 | $\frac{-6}{-1}+1=7$ 2 | ERR 3 | $\frac{-6}{1}+1=-5$ 4 | $\frac{-6}{2}+1=-2$	
HA:	Range:		
$y=1$	$\mathbb{R}, y \neq 1$		

14. $f(x) = \dfrac{2x}{x+5}$

VA:	Domain:	Table	Graph
$x+5=0$ $x=-5$	$\mathbb{R}, x \neq -5$	x | y -7 | $\frac{-14}{-2}$ -6 | $\frac{-12}{-1}$ -5 | ERR -4 | $\frac{-8}{1}$ -3 | $\frac{-6}{2}$	
HA:	Range:		
Same Degree $y=\frac{3}{1}$	$\mathbb{R}, y \neq 3$		

15. $f(x)=\frac{3x^2-3}{x^2-9}$ = $\frac{3(x^2-1)}{(x+3)(x-3)}$ = $\frac{3(x+1)(x-1)}{(x+3)(x-3)}$

VA:

$(x+3)=0$

$x=-3$

$x-3=0$

$x=3$

Domain:

$\mathbb{R}, x\neq 3, -3$

HA:

Same Degree

$y=\frac{3}{1}$

$y=3$

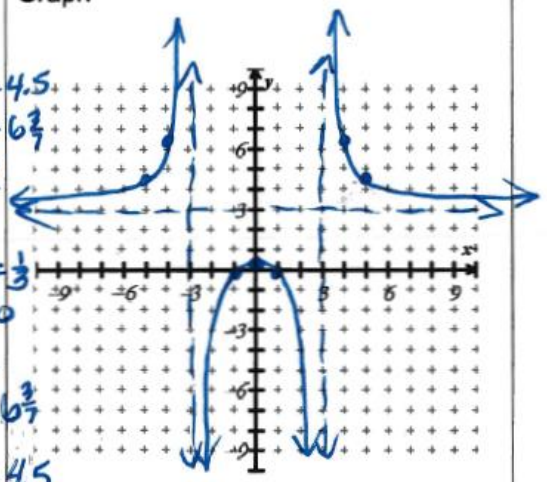
Range:

$\mathbb{R}, y\neq 3$

Table

x	y
-5	$72/16=4.5$
-4	$45/7=6\frac{3}{7}$
-3	ERR
-1	$0/-8$
0	$-3/-9=\frac{1}{3}$
1	$0/-8=0$
3	ERR
4	$45/7=6\frac{3}{7}$
5	$72/16=4.5$

Graph



16. $f(x)=\frac{x^2-9}{x^2+8x+15}$ = $\frac{(x+3)(x-3)}{(x+3)(x+5)}$ = $\frac{x-3}{x+5}$

VA:

$x+5=0$

$x=-5$

Domain:

$\mathbb{R}, x\neq -5$

HA:

Same Degree

$y=\frac{1}{1}$

$y=1$

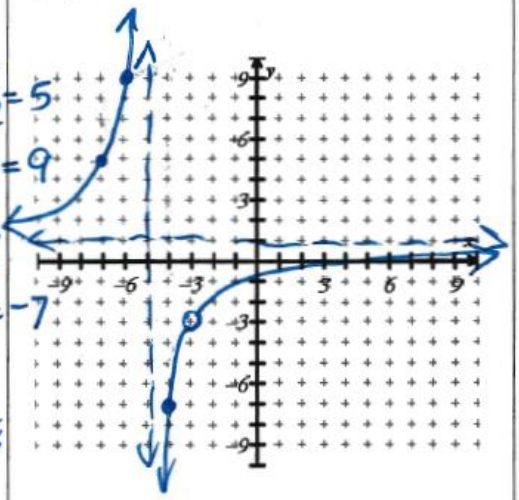
Range:

$\mathbb{R}, x\neq 1$

Table

x	y
-7	$-10/-2=5$
-6	$-9/-1=9$
-5	ERR
-4	$-7/1=-7$
-3	-3
	HOLE

Graph



Hole: $x+3=0$ $x=-3$ $y=\frac{-3-3}{-3+5}$ $y=\frac{-6}{2}=-3$ $(-3,-3)$

VARIATION

VARIATION:

1. Direct variation formula: $y = kx$
2. Inverse variation formula: $y = \frac{k}{x}$
3. Joint variation formula: $z = kxy$

Determine if the equation or situation represents direct, inverse, or joint variation, or neither.

4. $d = kst$ — joint
5. $m = \frac{k}{r}$ — inverse
6. $s = kpr$ — joint
7. $\frac{a}{b} = 5$ — Direct
8. $y = \frac{x}{10}$ — Direct
9. $y = x - 7$ — Neither

Translate each situation into an equation. (Do not solve!)

10. An equation shows m is directly proportional to n and inversely proportional to s cubed. When m = 5, then n = 160 and s = 2. What is the constant of proportionality? Write your answer as a fraction.

$$n = \frac{K}{s^3}$$

11. The weight, w, that a column of a bridge can support varies directly as the fourth power of its diameter, d, and inversely as the square of its length, l.

$$w = \frac{Kd^4}{l^2}$$

12. The number, n, of grapefruit that can fit into a box is inversely proportional to the cube of the diameter, d, of each grapefruit.

$$n = \frac{K}{d^3}$$

13. An equation shows m is directly proportional to n and inversely proportional to s cubed. When m = –4, then n = 160 and s = 2. What is the constant of proportionality?

$m = \frac{Kn}{s^3}$

$-4 = \frac{K(160)}{2^3}$

$-4 = \frac{160K}{8}$

$-32 = 160K$

$K = 5$

14. The variable z varies jointly with x and y. Write an equation relating x, y, and z when x = -4, y = 3, and z = 2.

$z = Kxy$

$2 = K(-4)(3)$

$\frac{2}{-12} = \frac{-12K}{-12}$

$K = -\frac{1}{6}$

Eq: $z = -\frac{1}{6}xy$

15. The amount of money earned at your job (m) varies directly with the number of hours (h) you work. The first day of work you earned $57 after working 6 hours. You are trying to save money to go to buy a new car to take to college next year. How many hours will you need to work in order to save $ 4750? = m

$m = Kh$

$\frac{57}{6} = \frac{K(6)}{6}$

$K = 9.5$

Eq: $m = 9.5h$

$\frac{4750}{9.5} = \frac{9.5h}{9.5}$

$500 = h$

500 hours

16. The force needed to keep a car from skidding on a curve varies jointly as the weight of the car and the square of the speed and inversely as the radius of the curve. Suppose a 3,960 lb. force is required to keep a 2,200 lb. car traveling at 30 mph from skidding on a curve of radius 500 ft. How much force is required to keep a 3,000 lb. car traveling at 45 mph from skidding on a curve of radius 400 ft.?

$F = \frac{Kws^2}{r}$

$3960 = \frac{K(2200)(30)^2}{500}$

$\frac{3960}{3960} = \frac{3960K}{3960}$

$K = 1$

Eq: $F = \frac{1ws^2}{r}$

$F = \frac{1(3000)(45)^2}{400}$

$F = 15,187.5$ lb