I. Model Problems

The equation of a line is given by the formula $y = mx + b$.
m equals the slope of the line
b equals the y-intercept of the line
This equation of the line is called **"slope-intercept" form** because it easily shows both the slope and the intercept of the line.

To find the equation of a line given the slope and intercept, simply plug into the equation.

Example 1 Write the equation of the line with slope 2 that has y-intercept 5.

$y = mx + b$	Write the slope-intercept formula.
$y = 2x + 5$	Substitute $m = 2$ and $b = 5$

The answer is $y = 2x + 5$.

To find the equation of a line given the slope and one point on the line, plug in the slope and the coordinates of the point to solve for b, the y-intercept.

Example 2 Write the equation of the line with slope 3 that passes through the point (-1, 6).

$y = mx + b$	Write the slope-intercept formula
$6 = 2(-1) + b$	Substitute $m = 2$ and $(x, y) = (-1, 6)$
$6 = -2 + b$	Simplify
$b = 8$	Add 2 to each side to solve for b
$y = 3x + 8$	Substitute $m = 3$ and $b = 8$ into the slope-intercept formula

The answer is $y = 3x + 8$.

Sometimes the slope of the equation is not given. To find the equation of a line that passes through two points, you must first calculate the slope, then follow the steps in Example 2.

Example 3 Write the equation of the line that passes through the points (3, -2) and (-2, 8).

$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ — Write the slope formula

$m = \frac{8-(-2)}{-2-3} = \frac{10}{-5} = -2$ — Substitute $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (8, -2)$

$y = mx + b$ — Write the point-slope form

$3 = -2(-2) + b$ — Substitute $m = -2$ and $(x, y) = (-2, 3)$.

$3 = 4 + b$ — Simplify.

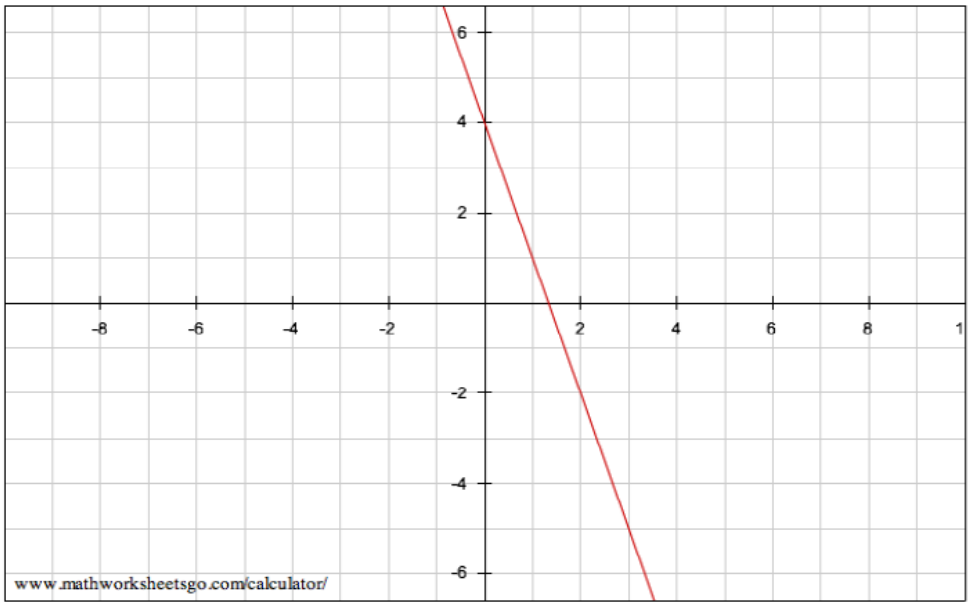
$b = -1$ — Subtract 4 from each side.

$y = -2x - 1$ — Substitute $m = -2$ and $b = -1$ into the point-slope formula.

The answer is $y = -2x - 1$.

Sometimes you will need to find the equation of a line given its graph.

Example 4 Write the equation of the line graphed below.



Notice that the graph passes through the points (0, 4) and (2, -2). The y-intercept is 4. This is the value of b.

$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ Write the slope formula

$m = \frac{4-(-2)}{0-2} = \frac{6}{-2} = -3$ Substitute $(x_1, y_1) = (2, -2)$ and $(x_2, y_2) = (0, 4)$

$y = mx + b$ Write the point-slope form

$y = -3x + 4$ Substitute $m = -3$ and $b = 4$ into the point-slope formula.

Practice

Find the equation of the line that has given slope and y-intercept.

1. $m = 2$ and $b = 7$

2. $m = -3$ and $b = 10$

3. $m = 10$ and $b = -3$

4. $m = -7$ and $b = 11$

5. $m = 4$ and $b = -20$

6. $m = -12$ and $b = -8$

7. $m = 6$ and $b = 6$

8. $m = -5$ and $b = -10$

Find the equation of the line with the given slope that passes through the given point.

9. $m = 2$ and (-1, 5)

10. $m = -4$ and (1, 1)

11. $m = -2$ and (-2, -2)

12. $m = 6$ and (2, 0)

13. $m = 3$ and (0, 7)

14. $m = -1$ and (4, 5)

15. $m = 1$ and (-2, 5)

16. $m = 0$ and (10, 7)

Find the equation of the line that passes through the given points.

17. (1, 2) and (-1, 5)

18. (-7, -7) and (-1, 4)

19. (1, 8) and (-3, 4)

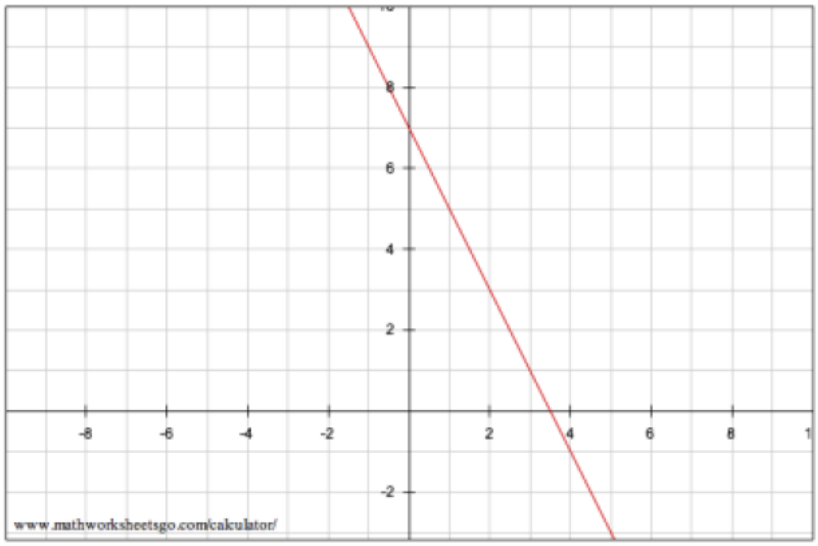
20. (1, 5) and (2, 0)

21. (6, 10) and (2, 8)

22. (-8, 4) and (2, -1)

Find the equation of each line graphed below.

23.



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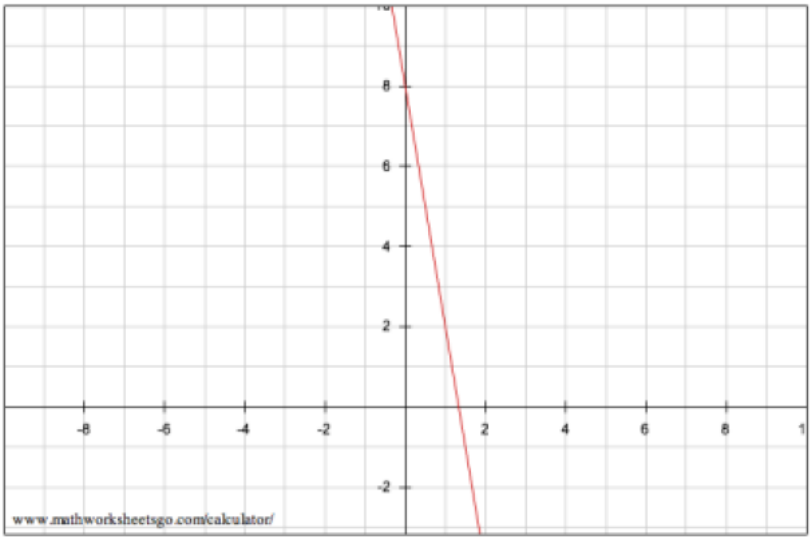

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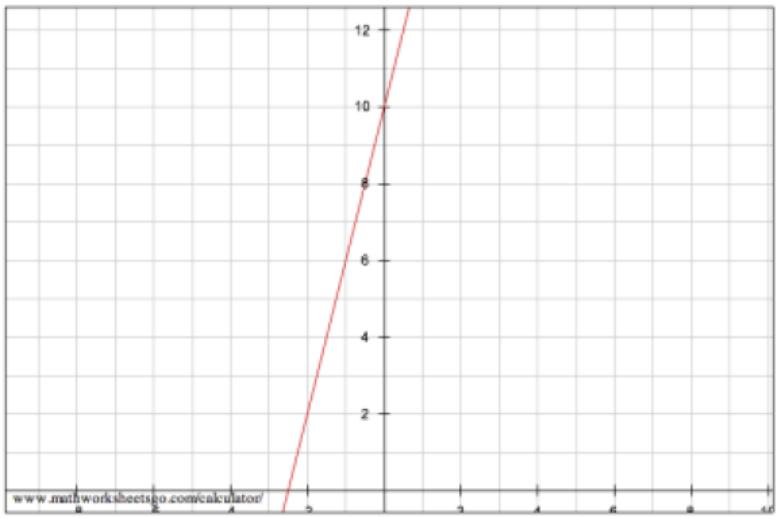

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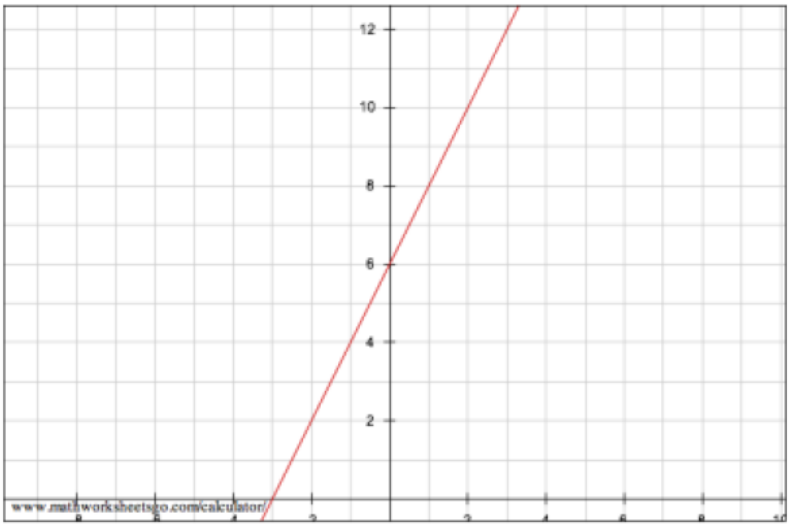

III. Challenge Problems

27. Explain why you cannot use $y = mx + b$ to find the equation of a vertical line.

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28. What is the equation of a line that passes through the points (-0.72, 1.42) and (4.22, 5.83)?

29. Correct the Error

There is an error in the student work shown below:

Question: Find the equation of the line that passes through the points (-1, 4) and (2, 7).

Solution:

The slope is given by the formula rise over run.

$$= \frac{7 - 4}{2 - (-1)} = \frac{3}{3} = 1$$

Plug into $y = mx + b$;

$y = mx + 1$.

Substitute (-1, 4) to solve for m:

$4 = -1 \bullet m + 1$ so $m = -3$

The equation of the line is $y = -3x + 1$.

What is the error? Explain how to solve the problem.

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Answers

1. $y = 2x + 7$
2. $y = -3x + 10$
3. $y = 10x - 3$
4. $y = -7x + 11$
5. $y = 4x - 20$
6. $y = -12x - 8$
7. $y = 6x + 6$
8. $y = -5x - 10$
9. $y = 2x + 7$
10. $y = -4x + 5$
11. $y = -2x + 6$
12. $y = 6x - 12$
13. $y = 3x + 7$
14. $y = -x + 9$
15. $y = x + 7$
16. $y = 7$
17. $y = -1.5x + 3.5$
18. $y = 1.833x + 5.833$
19. $y = x + 7$
20. $y = -5x + 10$
21. $y = 0.5x + 7$
22. $y = -0.5x$
23. $y = -2x + 7$
24. $y = -6x + 8$
25. $y = 4x + 10$
26. $y = 2x + 6$
27. The equation of a vertical line is an equation in the form $x =$ a constant. Vertical lines have infinite slope and typically do not have a y-intercept.
28. $y = 0.893x + 2.06$
29. The student switched the y-intercept and the slope in the equation of a line formula (the student mistakenly thought b was the slope)