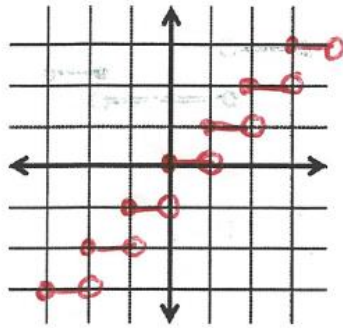


Step Functions

* The **[FLOOR]** function rounds down to the greatest integer less than or equal to the number.
Also known as the Greatest Integer Function

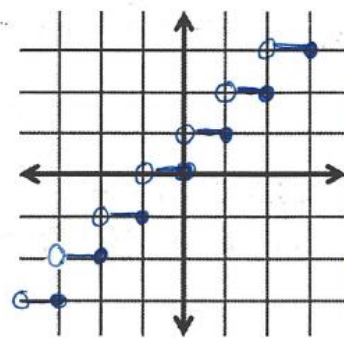
* The **[CEILING]** function rounds up to the lowest integer greater than or equal to the number.

$$\text{floor}(x) = \lfloor x \rfloor$$



Greatest Integer Function
 $f(x) = \lfloor x \rfloor$

$$\text{ceiling}(x) = \lceil x \rceil$$



X	Y
0	0
0.1	1
0.2	1
⋮	⋮
0.999	1
1	1
1.1	2
1.2	2
⋮	⋮
1.999	2
2	2
2.1	3
2.2	3
⋮	⋮

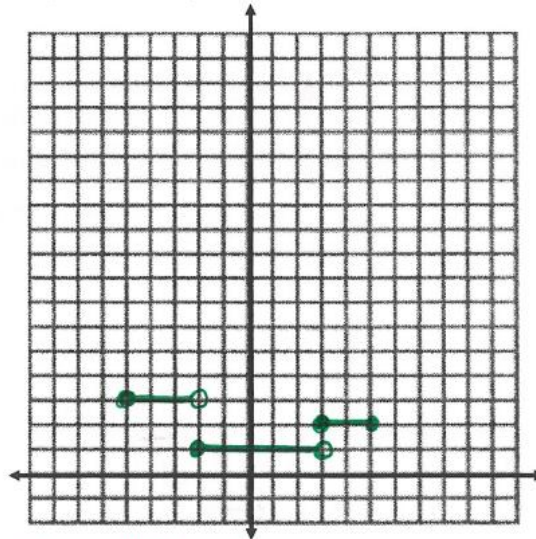
X	Y
0	0
0.1	0
0.2	0
⋮	⋮
0.999	0
1	1
1.1	1
1.2	1
⋮	⋮
1.999	1
2	2
2.1	2
2.2	2
⋮	⋮
2.999	2

1) Complete the following table.

x	$\lfloor x \rfloor$ or $\lfloor x \rfloor$	$\lceil x \rceil$
4.8	4	5
-1.3	-2	-1
2.2	2	3
6	6	6
-3	-3	-3
-2/3	-1	0
π	3	4

2) The following piecewise function is an example of a step-function.

$$S(x) = \begin{cases} 3 & -5 \leq x < -2 \\ 1 & -2 \leq x < 3 \\ 2 & 3 \leq x \leq 5 \end{cases}$$



a) Graph this function and state the domain and range.

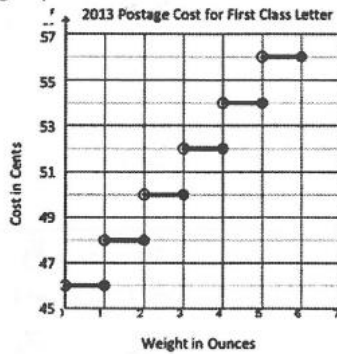
$D: [-5, 5]$

$R: \{1, 2, 3\}$

b) Why is this type of function called a step function?

The horizontal line segments step up & down like "Steps"

3) Does the following graph show a floor or ceiling function?



ceiling function $\lceil x \rceil$

Weight in ounces, x	2	2.2	3	3.5	4
Cost of postage, $C(x)$	48	50	50	52	52

a) Complete the table.

b) If it costs Trina \$0.54 to mail her letter, how many ounces did it weigh?

it weighed more than 4oz., but less than or equal to 5oz.

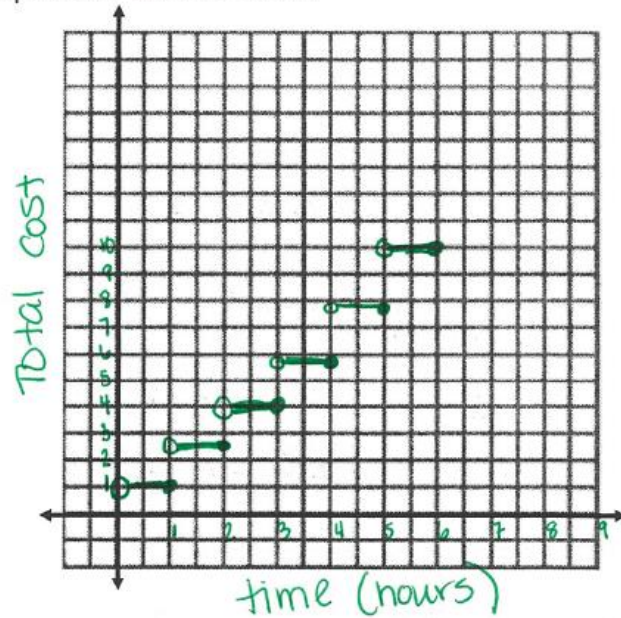
c) Write a formula involving step functions that represents the cost of postage based on the graph shown above.

$f(x) = 2 \lceil x \rceil + 44, 0 < x \leq 6$

4) The table below lists the total cost for parking for a period of time on a street in Albany, N.Y. The total cost is for any length of time up to and including the hours parked. For example, parking for up to and including 1 hour would cost \$1.25; parking for 3.5 hours would cost \$5.75.

a) Graph the step function that would represent this situation.

Hours Parked	Total Cost	(*)
1	1.25	↙ 1.25
2	2.50	↙ 1.50
3	4.00	↙ 1.75
4	5.75	↙ 2.00
5	7.75	↙ 2.25
6	10.00	



b) Write it as a piecewise function.

$$f(x) = \begin{cases} 1.25, & 0 < x \leq 1 \\ 2.50, & 1 < x \leq 2 \\ 4, & 2 < x \leq 3 \\ 5.75, & 3 < x \leq 4 \\ 7.75, & 4 < x \leq 5 \\ 10, & 5 < x \leq 6 \end{cases}$$

c) Explain how the cost per hour to park changes over the six-hour period.

The cost increases by \$0.25 (*) for each additional hour.

5) Jim is parking his car at the airport while he goes on a trip. He looked up the rates for the parking garage:

Garage Parking Rates	
Located on floors two, three, four and five of the parking garage	
First Hour:	\$2.00
Each Additional Hour:	\$2.00
* Maximum Daily Rate:	\$12.00
Five Consecutive Days:	\$50.00
Seven Consecutive Days:	\$64.00

time	cost	time	cost
1	2	6 to 24	12
2	4	2 days	24
3	6	3 days	36
4	8	4 days	48
5	10	5-6 days	50
6	12	7 days	64

Max Daily rate *

a) Write a piecewise function that models this situation. Use step functions!

$$f(x) = \begin{cases} 2\lceil x \rceil, & 0 < x \leq 6 \\ 12\lceil \frac{x}{24} \rceil, & 6 < x < 120 \\ 50, & 120 \leq x \leq 168 \\ 64, & 128 \leq x < 192 \end{cases}$$

b) How much would it cost him if he parked his car for 4 hours? 2 days?

4 hours: \$8 2 days: \$24

Sum It Up!

- Piecewise functions can be used to model many real life situations.
- The floor and ceiling function are both examples of step functions.
- A step function has a constant output and resembles steps when graphed.