Perform Function Operations and Composition

Operations of Functions

Let ***f*** and ***g*** be any two functions. A new function ***h*** can be defined by performing any of the four basic operations on ***f*** and ***g***.

Operation	Definition	Example: $f(x)=5x,\ g(x)=x+2$
Addition	$h(x)=f(x)+g(x)$ OR $h(x)=(f+g)(x)$	
Subtraction	$h(x)=f(x)-g(x)$ OR $h(x)=(f-g)(x)$	
Multiplication	$h(x)=f(x)\bullet g(x)$ OR $h(x)=(f\bullet g)(x)$	
Division	$h(x)=\frac{f(x)}{g(x)}$ OR $h(x)=\left(\frac{f}{g}\right)(x)$	

Given $f(x) = 2x^2 - 5x - 3$ and $g(x) = x^2 - 4x + 3$, perform the indicated operations.

1. $(f+g)(x)$	2. $(f-g)(x)$
3. $g(x) - f(x)$	4. $(g \bullet f)(x)$
5. $\frac{f(x)}{g(x)}$	6. $\left(\frac{g}{f}\right)(x)$

Composition of Functions: the process of combining two or more functions to create a new function. It is a process through which we will substitute an entire function into another function.

The **composition** of a function g with a function f is $h(x) = g(f(x))$, which can also be written as $[g \circ f](x)$. This is read aloud as "g of f of x".

To Evaluate $g(f(x))$:

1. Find the $f(x)$ value (the output).
2. Put the $f(x)$ value into $g(x)$ to find $g(f(x))$.

To write the new rule g (f (x)):

1. Plug in the rule of f (x) into every x in g(x).
2. Simplify.

** The *OUTPUT* of $f(x)$ becomes the *INPUT* of $g(x)$. **

Given $f = \{(1,2)\ (3,4)\ (5,4)\}$ and $g = \{(2,3)\ (4,3)\ (6,1)\}$, evaluate the composition.

7. $[f \circ g](2)$	8. $g(f(3))$	9. $[g \circ f](1)$	10. $[f \circ g](6)$

Given $f(x) = x + 7$, $g(x) = x^2 - 3x + 6$, $h(x) = 4x - 1$, evaluate or perform the composition.

11. $[h \circ g](2)$	12. $h(f(-10))$	13. $h(g(x))$
14. $[g \circ h](x)$	15. $h(f(g(x)))$	

Function Operations

Domain & Range

Domain: Possible values for ____ that you may put into a function

The domain will be ________________UNLESS: (only 2 exceptions!!!)

- X is in a **Denominator**
  - DENOMINATOR CANNOT BE ________. Specifically $x \neq 0$
- X is under an **Even Radical**
  - THE RADICAND CANNOT BE ________. Therefore $x \geq 0$

Ex 1 Perform the operation given that f(x) = $4x^{\frac{1}{2}}$ and g(x) = $-9x^{\frac{1}{2}}$. State the domain of the resultant function.

a) f(x) + g(x)	b) f(x) – g(x)
Domain:________________________	Domain:________________________

Division:

1) **Plug in the functions**
2) **Factor each one separately**
3) **Cancel out any common factors**

EVERYTHING in the parentheses must cancel- not just one part

Ex 2 Perform the operation given that f(x) = $4x^2 + 8x$ and g(x) = x + 2. State the domain of the resultant function.

a) $\frac{g(x)}{f(x)}$	b) $\frac{f(x)}{g(x)}$
Domain:________________________	Domain:________________________

Ex 3 Perform the operations given that $f(x) = 2x$ and $g(x) = 3x^2 - 5$.

c) $f(g(x))$ $f(g(x)) =$ ____________ Domain:____________	d) $g \circ f(x)$ $g \circ f(x) =$ ____________ Domain:____________

Ex 4 State the domain of the following functions.

a. $f(x) = \frac{4}{x-3}$	e. $g(x) = \frac{5}{x-9}$
b. $h(x) = \frac{5}{2x+7}$	f. $j(x) = \frac{9}{x^2-4}$
c. $k(x) = \sqrt{2x}$	g. $l(x) = \sqrt{2-3x}$
d. $d(x) = \frac{1}{\sqrt{1-x}}$	h. $g(k(x))$

Perform Function Operations and Composition

Operations of Functions (resultant)

Let f and g be any two functions. A new function h can be defined by performing any of the four basic operations on f and g.

Operation	Definition	Example: $f(x)=5x,\ g(x)=x+2$
Addition	$h(x)=f(x)+g(x)$ OR $h(x)=(f+g)(x)$	$h(x)=5x+x+2$ $h(x)=6x+2$
Subtraction	$h(x)=f(x)-g(x)$ OR $h(x)=(f-g)(x)$	$h(x)=5x-(x+2)$ $h(x)=5x-x-2$ $h(x)=4x-2$
Multiplication	$h(x)=f(x)\bullet g(x)$ OR $h(x)=(f\bullet g)(x)$	$h(x)=5x\cdot(x+2)$ $h(x)=5x^2+10x$
Division	$h(x)=\frac{f(x)}{g(x)}$ OR $h(x)=\left(\frac{f}{g}\right)(x)$	$h(x)=\frac{5x}{x+2}$

Given $f(x)=2x^2-5x-3$ and $g(x)=x^2-4x+3$, perform the indicated operations.

1. $(f+g)(x) = 2x^2-5x-3+x^2-4x+3$ $f(x)+g(x)=3x^2-9x$	2. $(f-g)(x)=2x^2-5x-3-(x^2-4x+3)$ $f(x)-g(x)=2x^2-5x-3-x^2+4x-3$ $f(x)-g(x)=x^2-x-6$
3. $g(x)-f(x)=x^2-4x+3-(2x^2-5x-3)$ $g(x)-f(x)=x^2-4x+3-2x^2+5x+3$ $g(x)-f(x)=-x^2+x+6$	4. $(g\bullet f)(x)$ $g(x)\cdot f(x)=(x^2-4x+3)(2x^2-5x-3)$ $=2x^4-5x^3-3x^2-8x^3+20x^2+12x+6x^2-15x-9$ $g(x)\cdot f(x)=2x^4-13x^3+23x^2-3x-9$
5. $\frac{f(x)}{g(x)}=\frac{2x^2-5x-3}{x^2-4x+3}=\frac{(x-3)(2x+1)}{(x-3)(x-1)}$ $\frac{f(x)}{g(x)}=\frac{2x+1}{x-1}$	6. $\left(\frac{g}{f}\right)(x)=\frac{x^2-4x+3}{2x^2-5x-3}=\frac{(x-3)(x-1)}{(x-3)(2x+1)}$ $\frac{g(x)}{f(x)}=\frac{(x-1)}{(2x+1)}$

$2x^2-6x+x-3$
$2x(x-3)+1(x-3)$

Composition of Functions: the process of combining two or more functions to create a new function. It is a process through which we will substitute an entire function into another function.

The **composition** of a function g with a function f is $h(x) = g(f(x))$, which can also be written as $[g \circ f](x)$. This is read aloud as "g of f of x".

To write the new ~~rule~~ resultant function **$g(f(x))$:**

1. Plug in the rule of $f(x)$ into every x in $g(x)$.
2. Simplify.

To **Evaluate** $g(f(x))$:

1. Find the $f(x)$ value (the output).
2. Put the $f(x)$ value into $g(x)$ to find $g(f(x))$.

** The *OUTPUT* of $f(x)$ becomes the *INPUT* of $g(x)$. **

Given $f = \{(1,2)\ (3,4)\ (5,4)\}$ and $g = \{(2,3)\ (4,3)\ (6,1)\}$, **evaluate the composition.**

7. $[f \circ g](2)$ $f(g(2))$ $g(2)=3$ $f(3)=4$ $f(g(2))=4$	8. $g(f(3))$ $f(3)=4$ $g(4)=3$ $g(f(3))=3$	9. $[g \circ f](1)$ $g(f(1))$ $f(1)=2$ $g(2)=3$ $g(f(1))=3$	10. $[f \circ g](6)$ $f(g(6))$ $g(6)=1$ $f(1)=2$ $f(g(6))=2$

Given $f(x) = x + 7$, $g(x) = x^2 - 3x + 6$, $h(x) = 4x - 1$, **evaluate or perform the composition.**

11. $[h \circ g](2)$ $h(g(2))$ $g(2)=2^2-3(2)+6$ $g(2)=4-6+6=4$ $h(4)=4(4)-1=15$ $h(g(2))=15$	12. $h(f(-10))$ $f(-10)=-10+7=-3$ $h(-3)=4(-3)-1$ $=-12-1=-13$ $h(f(-10))=-13$ $f(☺)=☺+7$ oBrw	13. $h(g(x))$ $h(x^2-3x+6)$ $=4(x^2-3x+6)-1$ $=4x^2-12x+24-1$ $h(g(x))=4x^2-12x+23$

14. $[g \circ h](x)$ $g(h(x))=g(4x-1)$ $=(4x-1)^2-3(4x-1)+6$ $=16x^2-8x+1-12x+3+6$ $g(h(x))=16x^2-20x+10$	✱ 15. $h(f(g(x)))$ $h(f(x^2-3x+6))$ $h(x^2-3x+6+7)$ $h(x^2-3x+13)$ $4(x^2-3x+13)-1$ $4x^2-12x+52-1$ $h(f(g(x)))=4x^2-12x+51$

Function Operations

Division, Domain & Range

Domain: Possible values for X that you may put into a function

The domain will be $(-\infty, \infty)$ $\mathbb{R}, x \in \mathbb{R}$ UNLESS: (only 2 exceptions!!!)

- X is in a **Denominator**
  - DENOMINATOR CANNOT BE zero. Specifically $x \neq 0$
- X is under an **Even Radical**
  - THE RADICAND CANNOT BE < 0. Therefore $x \geq 0$

Ex 1 Perform the operation given that $f(x) = 4x^{\frac{1}{2}}$ and $g(x) = -9x^{\frac{1}{2}}$. State the domain of the resultant function.

$f(x) = 4\sqrt{x} \quad g(x) = -9\sqrt{x}$

a) $f(x) + g(x)$	b) $f(x) - g(x)$
$4\sqrt{x} + (-9\sqrt{x})$	$4\sqrt{x} - (-9\sqrt{x})$
$f(x) + g(x) = -5\sqrt{x}$	$f(x) - g(x) = 13\sqrt{x}$
Domain: $x \geq 0$ or $[0, \infty)$	Domain: $x \geq 0$ or $[0, \infty)$

Division:

1) **Plug in the functions**
2) **Factor each one separately**
3) **Cancel out any common factors**

EVERYTHING in the parentheses must cancel- **not just one part**

Ex 2 Perform the operation given that $f(x) = 4x^2 + 8x$ and $g(x) = x + 2$. State the domain of the resultant function.

a)	b)
$\frac{g(x)}{f(x)} = \frac{x+2}{4x^2+8x} = \frac{(x+2)}{4x(x+2)}$	$\frac{f(x)}{g(x)} = \frac{4x^2+8x}{x+2} = \frac{4x(x+2)}{(x+2)}$
$\frac{g(x)}{f(x)} = \frac{1}{4x}$ $x \neq 0$, $x+2 \neq 0$, $x \neq -2$	$\frac{f(x)}{g(x)} = 4x$ $x+2 \neq 0$
Domain: $x \neq -2, x \neq 0$ $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$	Domain: $x \neq -2$ $(-\infty, -2) \cup (-2, \infty)$

Ex 3 Perform the operations given that $f(x) = 2x$ and $g(x) = 3x^2 - 5$.

c) $f(g(x)) = f(3x^2-5)$ $= 2(3x^2-5)$ $f(g(x)) =$ $6x^2 - 10$ Domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$	d) $g \circ f(x)$ $g(f(x)) = g(2x)$ $= 3(2x)^2 - 5$ $= 3(4x^2) - 15$ $g \circ f(x) =$ $12x^2 - 15$ Domain: $x \in \mathbb{R}$ or $(-\infty, \infty)$

Ex 4 State the domain of the following functions.

a. $f(x) = \frac{4}{x-3}$ $x - 3 \neq 0$ D: $x \neq 3$ or D: $(-\infty, 3) \cup (3, \infty)$	e. $g(x) = \frac{5}{x-9}$ $x - 9 \neq 0$ D: $x \neq 9$ or D: $(-\infty, 9) \cup (9, \infty)$
b. $h(x) = \frac{5}{2x+7}$ $2x + 7 \neq 0$ D: $x \neq -\frac{7}{2}$ or D: $(-\infty, -\frac{7}{2}) \cup (-\frac{7}{2}, \infty)$	f. $j(x) = \frac{9}{x^2-4}$ $x^2 - 4 \neq 0$ $x^2 \neq 4$ D: $x \neq \pm 2$ or D: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
c. $k(x) = \sqrt{2x}$ $2x \geq 0$ D: $x \geq 0$ or D: $[0, \infty)$	g. $l(x) = \sqrt{2-3x}$ $2 - 3x \geq 0$ $-3x \geq -2$ D: $x \leq \frac{2}{3}$ or D: $(-\infty, \frac{2}{3}]$
d. $d(x) = \frac{1}{\sqrt{1-x}}$ $1 - x \geq 0$ $-x \geq -1$ $x \leq 1$ $\sqrt{1-x} \neq 0$ D: $x < 1$ $1 - x \neq 0$ D: $(-\infty, 1)$ $1 \neq x$	(*) h. $g(k(x))$ $g(\sqrt{2x})$ D: $x \geq 0$, $x \neq \frac{81}{2}$ $g(k(x)) = \frac{5}{\sqrt{2x} - 9}$ D: $[0, \frac{81}{2}) \cup (\frac{81}{2}, \infty)$

$2x \geq 0$ $\sqrt{2x} - 9 \neq 0$

$x \geq 0$ $\sqrt{2x} \neq 9$

$2x \neq 81$

$x \neq \frac{81}{2}$