Hence, we get another piecewise definition, depending on whether the index is even or odd:

$$\boxed{\sqrt[n]{x^n} = \begin{cases} x; \text{ if } n \text{ is odd} \\ |x|; \text{ if } n \text{ is even} \end{cases}}$$

Thus "Power First, Then Root" $\Longrightarrow$ cancel only if the index is odd; otherwise absolute value!

Properties of Radicals

$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

$$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Summary of Formulas

1. $\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$ UNLESS index is even with x possibly negative

2. $\boxed{x^{\frac{m}{n}} = (\sqrt[n]{x})^m}$ UNLESS index is even with x possibly negative

3. $\boxed{(\sqrt[n]{x})^m = \sqrt[n]{x^m}}$ UNLESS index is even with x possibly negative

4. Piecewise Definition of $|x|$:

$$\boxed{|x| = \begin{cases} x; \text{ if } x \geq 0 \\ -x; \text{ if } x < 0 \end{cases}}$$

5. $\boxed{(\sqrt[n]{x})^n = x}$ "Root First, Then Power" $\Longrightarrow$ CANCEL

6. Piecewise Definition of $\sqrt[n]{x^n}$:

$$\boxed{\sqrt[n]{x^n} = \begin{cases} x; \text{ if } n \text{ is odd} \\ |x|; \text{ if } n \text{ is even} \end{cases}}$$