

The Slope of a Line

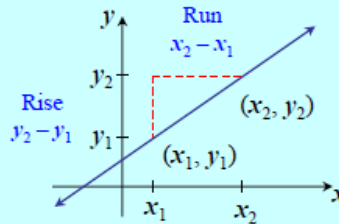
Mathematicians have developed a useful measure of the steepness of a line, called the **slope** of the line. Slope compares the vertical change (the **rise**) to the horizontal change (the **run**) when moving from one fixed point to another along the line. A ratio comparing the change in y (the rise) with the change in x (the run) is used calculate the slope of a line.

Definition of Slope

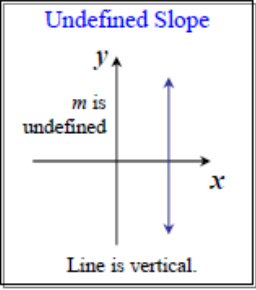
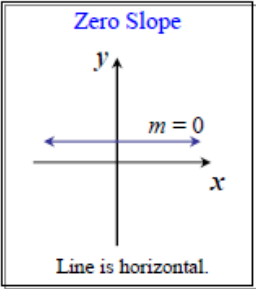
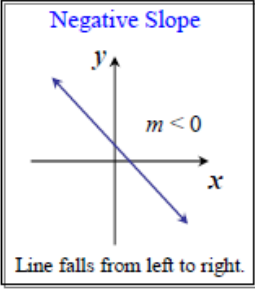
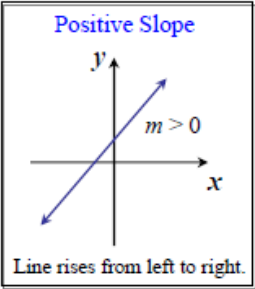
The slope of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_1 \neq x_2$.



The Possibilities for a Line's Slope



Point-Slope Form of the Equation of a Line

The **point-slope equation** of a non-vertical line of slope m that passes through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Slope-Intercept Form of the Equation of a

The **slope-intercept equation** of a non-vertical line with slope m and y -intercept b is

$$y = mx + b.$$

Equations of Horizontal and Vertical Lines

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y -intercept. Note: $m = 0$.

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a$$

where a is the x -intercept. Note: m is undefined.

General Form of the Equation of the a Line

Every line has an equation that can be written in the general form

$$Ax + By + C = 0$$

Where A , B , and C are three integers, and A and B are not both zero. A must be positive.

Standard Form of the Equation of the a Line

Every line has an equation that can be written in the standard form

$$Ax + By = C$$

Where A , B , and C are three integers, and A and B are not both zero. A must be positive.

In this form, $m = -A/B$ and the intercepts are $(0, C/B)$ and $(C/A, 0)$.

Equations of Lines

- Point-slope form: $y - y_1 = m(x - x_1)$
- Slope-intercept form: $y = m x + b$
- Horizontal line: $y = b$
- Vertical line: $x = a$
- General form: $Ax + By + C = 0$
- Standard form: $Ax + By = C$

Steps for Graphing $y = mx + b$

Graphing $y = mx + b$ by Using the Slope and y -Intercept

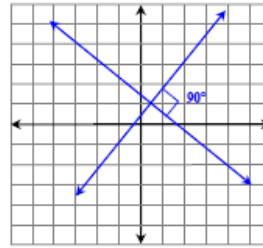
- Plot the y -intercept on the y -axis. This is the point $(0, b)$.
- Obtain a second point using the slope, m . Write m as a fraction, and use rise over run starting at the y -intercept to plot this point.
- Use a straightedge to draw a line through the two points. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Slope and Parallel Lines

- If two non-vertical lines are parallel, then they have the **same** slope.
- If two distinct non-vertical lines have the **same** slope, then they are parallel.
- Two distinct vertical lines, both with undefined slopes, are parallel.

Slope and Perpendicular Lines

Two lines that intersect at a right angle (90°) are said to be perpendicular. There is a relationship between the slopes of perpendicular lines.



Slope and Perpendicular Lines

- If two non-vertical lines are perpendicular, then the product of their slopes is -1 .
- If the product of the slopes of two lines is -1 , then the lines are perpendicular.
- A horizontal line having zero slope is perpendicular to a vertical line having undefined slope.