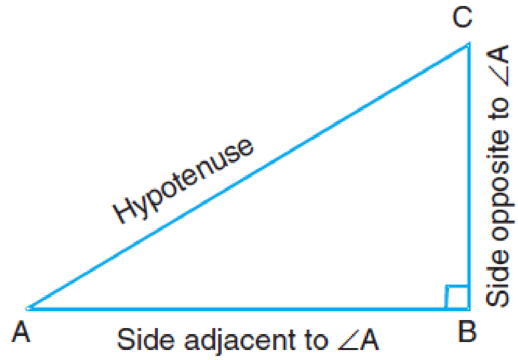
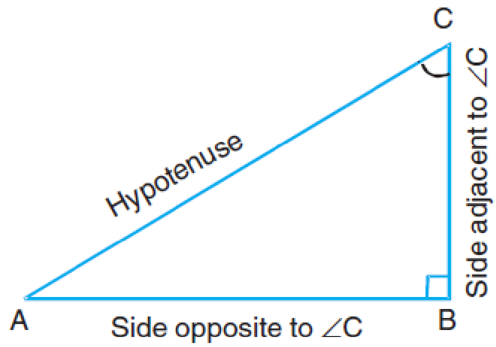
TRIGONOMETRIC RATIOS OF AN ACUTE ANGLE OF A RIGHT ANGLED TRIANGLE

Let there be a right triangle ABC, right angled at B. Here ∠A (i.e. ∠CAB) is an acute angle, AC is hypotenuse, side BC is opposite to ∠A and side AB is adjacent to ∠A.



Again, if we consider acute ∠C, then side AB is side opposite to ∠C and side BC is adjacent to ∠C.



We now define certain ratios involving the sides of a right triangle, called **trigonometric ratios**.

The trigonometric ratios of ∠A in right angled ΔABC are defined as:

(i) $\text{sine A} = \dfrac{\text{side opposite to } \angle A}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

(ii) $\text{cosine A} = \dfrac{\text{side adjacent to } \angle A}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

(iii) $\text{tangent A} = \dfrac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \dfrac{BC}{AB}$

(iv) $\text{cosecant A} = \dfrac{\text{Hypotenuse}}{\text{side opposite to } \angle A} = \dfrac{AC}{BC}$

(v) $\text{secant A} = \dfrac{\text{Hypotenuse}}{\text{side adjacent to } \angle A} = \dfrac{AC}{AB}$

(vi) $\text{cotangent A} = \dfrac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \dfrac{AB}{BC}$

The above trigonometric ratios are abbreviated as sin A, cos A, tan A, cosec A, sec A and cot A respectively. Trigonometric ratios are abbreviated as **t-ratios**.

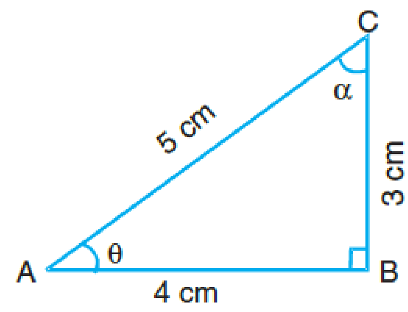
If we write $\angle A = \theta$, then the above results are

$$\sin\theta = \frac{BC}{AC}, \qquad \cos\theta = \frac{AB}{AC}, \qquad \tan\theta = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{AC}{BC}, \qquad \sec\theta = \frac{AC}{AB} \qquad \text{and} \quad \cot\theta = \frac{AB}{BC}$$

Note: Observe here that $\sin\theta$ and $\operatorname{cosec}\theta$ are reciprocals of each other. Similarly $\cot\theta$ and $\sec\theta$ are respectively reciprocals of $\tan\theta$ and $\cos\theta$.

Remarks



Thus in right ΔABC,

$AB = 4cm$, $BC = 3cm$ and

$AC = 5cm$, then

$$\sin \theta = \frac{BC}{AC} = \frac{3}{5}$$

$$\cos \theta = \frac{AB}{AC} = \frac{4}{5}$$

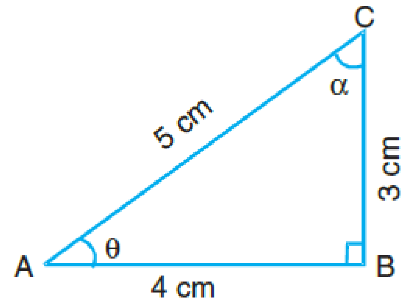
$$\tan \theta = \frac{BC}{AB} = \frac{3}{4}$$

$$\text{cosec } \theta = \frac{AC}{BC} = \frac{5}{3}$$

$$\sec \theta = \frac{AC}{AB} = \frac{5}{4}$$

and $$\cot \theta = \frac{AB}{BC} = \frac{4}{3}$$

In the above figure, if we take angle C = α, then



$$\sin \alpha = \frac{\text{side opposite to } \angle\alpha}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4}{5}$$

$$\cos \alpha = \frac{\text{side adjacent to } \angle\alpha}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{5}$$

$$\tan \alpha = \frac{\text{side opposite to } \angle\alpha}{\text{side adjacent to } \angle\alpha} = \frac{AB}{BC} = \frac{4}{3}$$

$$\operatorname{cosec} \alpha = \frac{\text{Hypotenuse}}{\text{side opposite to } \angle\alpha} = \frac{AC}{AB} = \frac{5}{4}$$

$$\sec \alpha = \frac{\text{Hypotenuse}}{\text{side adjacent to } \angle\alpha} = \frac{AC}{BC} = \frac{5}{3}$$

and $\cot \alpha = \dfrac{\text{side adjacent to } \angle\alpha}{\text{side opposite to } \angle\alpha} = \dfrac{BC}{AB} = \dfrac{3}{4}$

Remarks :

1. Sin A or sin θ is one symbol and sin cannot be separated from A or θ. It is not equal to sin × θ. The same applies to other trigonometric ratios.
2. Every t-ratio is a real number.
3. For convenience, we use notations $\sin^2\theta$, $\cos^2\theta$, $\tan^2\theta$ for $(\sin\theta)^2$, $(\cos\theta)^2$, and $(\tan\theta)^2$ respectively. We apply the similar notation for higher powers of trigonometric ratios.
4. We have restricted ourselves to t-ratios when A or θ is an acute angle.

Now the question arises: "Does the value of a t-ratio remains the same for the same angle of different right triangles?." To get the answer, let us consider a right triangle ABC, right angled at B. Let P be any point on the hypotenuse AC.

Let PQ $\perp$ AB

Now in right ΔABC,

$$\sin A = \frac{BC}{AC} \qquad \text{----(i)}$$

and in right ΔAQP,

$$\sin A = \frac{PQ}{AP} \qquad \text{----(ii)}$$

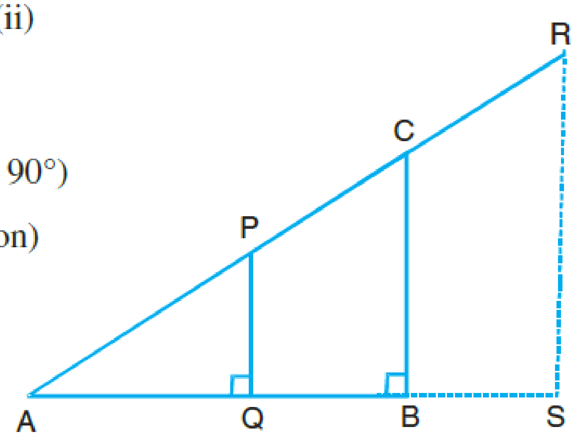


Fig. 22.5

Now in ΔAQP and ΔABC,

$$\angle Q = \angle B \qquad \text{----(Each = 90°)}$$

and $$\angle A = \angle A \qquad \text{----(Common)}$$

$\therefore$ $\Delta AQP \sim \Delta ABC$

$$\therefore \frac{AP}{AC} = \frac{QP}{BC} = \frac{AQ}{AB}$$

or $$\frac{BC}{AC} = \frac{PQ}{AP} \qquad \text{----(iii)}$$

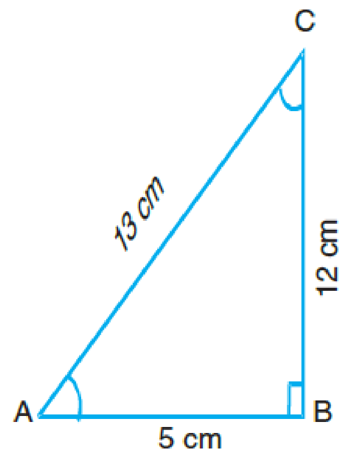
From (i), (ii), and (iii), we find that sin A has the same value in both the triangles.

Similarly, we have $\cos A = \frac{AB}{AC} = \frac{AQ}{AP}$ and $\tan A = \frac{BC}{AB} = \frac{PQ}{AQ}$

Let R be any point on AC produced. Draw RS $\perp$ AB produced meeing it at S. You can verify that value of t-ratios remains the same in ΔASR also.

Thus, we conclude that the value of trigonometric ratios of an angle does not depend on the size of right triangle. They only depend on the angle.

Example 22.1: In Fig. 22.6, ΔABC is right angled at B. If AB = 5 cm, BC = 12 cm and AC = 13 cm, find the value of tan C, cosec C and sec C.



Solution: We know that

$$\tan C = \frac{\text{side opposite to } \angle C}{\text{side adjacent to } \angle C} = \frac{AB}{BC} = \frac{5}{12}$$

$$\text{cosec } C = \frac{\text{Hypotenuse}}{\text{side opposite to } \angle C} = \frac{AC}{AB} = \frac{13}{5}$$

and $$\sec C = \frac{\text{Hypotenuse}}{\text{side adjacent to } \angle C} = \frac{AC}{BC} = \frac{13}{12}$$

Example 22.2 : Find the value of sin θ, cot θ and Sec θ from Fig. 22.7.

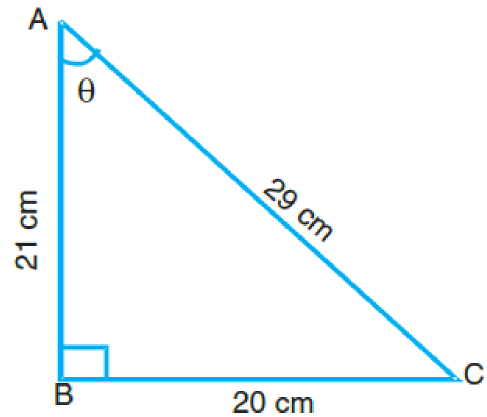


Fig. 22.7

Solution:

$$\sin\theta = \frac{\text{side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{20}{29}$$

$$\cot\theta = \frac{\text{side adjacent to } \angle\theta}{\text{side opposite to } \angle\theta} = \frac{AB}{BC} = \frac{21}{20}$$

and $$\sec\theta = \frac{\text{Hypotenuse}}{\text{side adjacent to } \angle\theta} = \frac{AC}{AB} = \frac{29}{21}$$

Example 22.4 : In Fig. 22.9, ΔABC is right angled at B, $\angle A = \angle C$, $AC = \sqrt{2}$ cm and AB = 1 cm. Find the values of sin C, cos C and tan C.

Solution: In ΔABC, $\angle A = \angle C$

$\therefore$ BC = AB = 1 cm (Given)

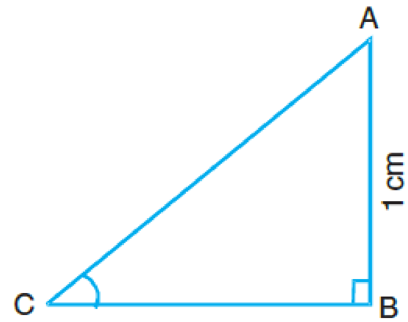


Fig. 22.9

$$\therefore \quad \sin C = \frac{\text{side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

and
$$\tan C = \frac{\text{side opposite to } \angle C}{\text{side adjacent to } \angle C} = \frac{AB}{BC} = \frac{1}{1} = 1$$

Remark: In the above example, we have $\angle A = \angle C$ and $\angle B = 90°$

$\therefore \ \angle A = \angle C = 45°$,

$\therefore$ We have $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

and $\tan 45° = 1$

Example 22.5 : In Fig. 22.10. ΔABC is right-angled at C. If AB = c, AC = b and BC = a, which of the following is true?

(i) $\tan A = \frac{b}{c}$

(ii) $\tan A = \frac{c}{b}$

(iii) $\cot A = \frac{b}{a}$

(iv) $\cot A = \frac{a}{b}$

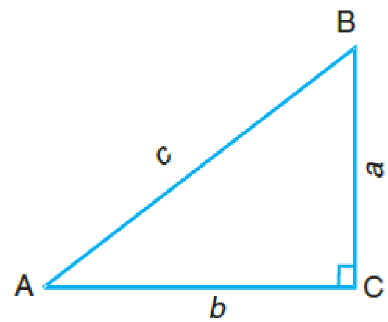


Fig. 22.10

Solution: Here $\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AC} = \frac{a}{b}$

and $\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{b}{a}$

Hence the result (iii) i.e. $\cot A = \frac{b}{a}$ is true.