Tangent and Cotangent Identities

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Reciprocal Identities

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} \qquad \tan(\theta) = \frac{1}{\cot(\theta)}$$

Even/Odd Formulas

$\sin(-\theta) = -\sin(\theta)$ $\csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$ $\sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\cot(-\theta) = -\cot(\theta)$

Periodic Formulas

If n is an integer then,

$$\sin(\theta + 2\pi n) = \sin(\theta) \quad \csc(\theta + 2\pi n) = \csc(\theta)$$

$$\cos(\theta + 2\pi n) = \cos(\theta) \quad \sec(\theta + 2\pi n) = \sec(\theta)$$

$$\tan(\theta + \pi n) = \tan(\theta) \quad \cot(\theta + \pi n) = \cot(\theta)$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Degrees to Radians

multiply by

$$\frac{\pi}{180}$$

Radians to Degrees

multiply by

$$\frac{180}{\pi}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \\ &= 1 - 2\sin^2(\theta)\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Half Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$$

Half Angle Formulas (alternate form)

$$\sin^2(\theta) = \tfrac{1}{2}\left(1-\cos(2\theta)\right)$$

$$\cos^2(\theta) = \tfrac{1}{2}\left(1+\cos(2\theta)\right)$$

$$\tan^2(\theta) = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos(\theta) \quad \cos\left(\frac{\pi}{2}-\theta\right)=\sin(\theta)$$

$$\csc\left(\frac{\pi}{2}-\theta\right)=\sec(\theta) \quad \sec\left(\frac{\pi}{2}-\theta\right)=\csc(\theta)$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot(\theta) \quad \cot\left(\frac{\pi}{2}-\theta\right)=\tan(\theta)$$

Inverse Trig Functions

Definition

$y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$

$y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$

$y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}(\cos(\theta)) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}(\sin(\theta)) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}(\tan(\theta)) = \theta$

Domain and Range

Function	Domain	Range
$y = \sin^{-1}(x)$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1}(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

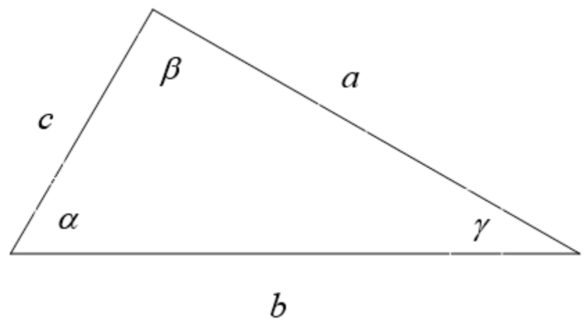
Alternate Notation

$\sin^{-1}(x) = \arcsin(x)$

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

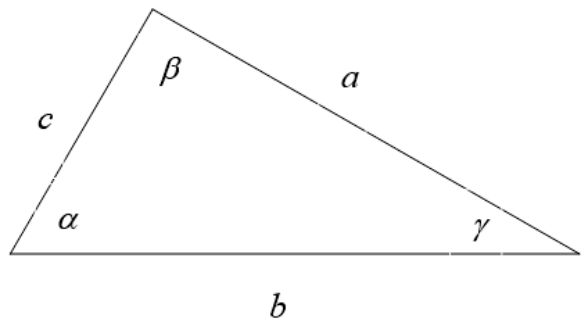
Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{1}{2}(\beta-\gamma)\right)}{\tan\left(\frac{1}{2}(\beta+\gamma)\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$

Law of Sines, Cosines and Tangents



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$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{1}{2}(\alpha-\gamma)\right)}{\tan\left(\frac{1}{2}(\alpha+\gamma)\right)}$$

Area of Triangle Law of Sines

Area of a Triangle
The area of any triangle is given by one half the product of the lengths of two sides times the sine of their included angle.

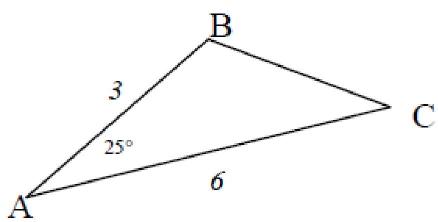
$$Area = \frac{1}{2}bc\sin A$$

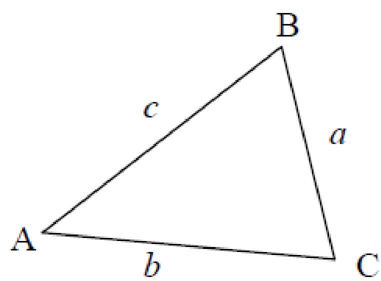
$$Area = \frac{1}{2}ab\sin C$$

$$Area = \frac{1}{2}ac\sin B$$

Find the area of the triangle.

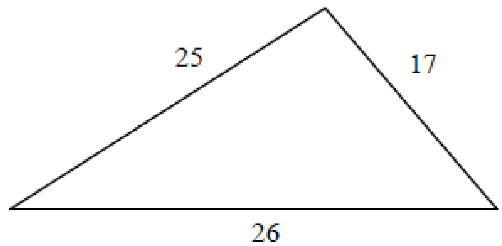
1.





How would you find the area of the given triangle using the most common area formula?

$A = \frac{1}{2}bh$



Since no height is given, it becomes quite difficult…

Heron's Formula

Heron's formula allows us to find the area of a triangle when only the lengths of the three sides are given. His formula states:

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

Where *a*, *b*, and *c*, are the lengths of the sides and *s* is the semiperimeter of the triangle.

Product to Sum Formulas

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$