Factoring — Quick Reference

Finding the GCF

Find the GCF for: $30x^3 + 5x^2 - 25x$

Step 1: Look at the coefficients

Ask yourself: Is there a number that I can divide 30, 5, and 25 by evenly? Yes, 5!
$(30/5 = 6)\ (5/5=1)\ (-25/5 = -5)$

Step 2: Look at the variable(s).

Ask yourself: Can I factor out a variable from EVERY term? Yes! Each term has at least one x, therefore, I can factor out x.

Step 3: Identify the GCF:
The GCF for this polynomial is: $5x$. (You can divide every term by 5x evenly (without creating a fraction).

Factoring Using the GCF

Factor the greatest common factor from:
$3x^4y^3 + 12x^3y - 18x^2y^2$

Solution:

Step 1: Look at the coefficients.
What is the GCF for 3, 12, 18?
$3x^4y^3 + 12x^3y - 18x^2y^2$
(What is the greatest number that can be divided into all evenly?)

3 is the GCF (for the coefficients).

Step 2: Look at the variable.
Can I factor out a variable for EVERY term? Yes! Each term contains at least one x^2 and y.(x^2y)
$3x^4y^3 + 12x^3y - 18x^2y^2$

Step 3: Identify the GCF.
The GCF is $3x^2y$. Now we are going to divide EVERY term by $3x^2y$. (Most students do this mentally, but I am going to write it out to show you the process.)

$$\frac{3x^4y^3}{3x^2y} + \frac{12x^3y}{3x^2y} - \frac{18x^2y^2}{3x^2y}$$

$x^2y^2 + 4x - 6y$ (the result after dividing)

Step 4: Write appropriately in factored form.

$3x^2y\,(x^2y^2 + 4x - 6y)$
GCF — Result after dividing each term by the GCF

Factored Form: $3x^2y\,(x^2y^2 + 4x - 6y)$

Factoring by Grouping

Factor the following polynomial:

$x^3y^3 + 2x^3 + 4x^2y^3 + 8x^2$

Step 1: **Separate** the polynomial into **2 or more groups** according to common factors. **Identify the common factor** for each group. (In this problem I need to rewrite the problem with common factors side by side.)

$x^3y^3 + 4x^2y^3$ $+ 2x^3 + 8x^2$

x^2y^3 is a common factor $2x^2$ is a common factor

Step 2: **Divide** each term by the common factor.

$$\frac{x^3y^3}{x^2y^3} + \frac{4x^2y^3}{x^2y^3} + \frac{2x^3}{2x^2} + \frac{8x^2}{2x^2}$$

$x + 4$ $+$ $x + 4$ ← Result after dividing

Step 3: Write appropriately in factored form.

$x^2y^3\,(x+4)$ $+$ $2x^2\,(x+4)$
Common Factor — Result after dividing — Common factor — Result after dividing

$x^2y^3\,(x+4) + 2x^2\,(x+4)$

Step 4: Now these two terms have a common factor. The common factor is (x+4). We can factor (x+4) and we are left with:

$(x+4)(x^2y^3 + 2x^2)$ This is the final answer in factored form.

Factoring Trinomials

Factor: $x^2 - 10x + 21$

Step 1: We are going to form two binomials, so write two sets of parenthesis.

$(\quad)\ (\quad)$

Step 2: What can we multiply together to get x^2? (The first term of each binomial is multiplied together to get the first term of the trinomial. $(x \cdot x = x^2)$

$x^2 - 10x + 21$

$(x\quad)\ (x\quad)$

Step 3: We need to find **two numbers** that we can **add** together to get -10 AND **multiply** together to get 21. (You must remember to take the sign in front of the term with it; therefore, the middle term is -10).

$x^2 - 10x + 21$

Two numbers that we **add** to get -10 & **multiply** to get 21.

In order to add two numbers together and get a negative number and then multiply the same two numbers and get a positive number, both numbers must be negative.*

Factors of 21	Sum of Factors
$-1 \cdot -21 = 21$	$-1 + (-21) = -22$
$-3 \cdot -7 = 21$	$-3 + (-7) = -10$

(There are other factors, but I will stop here since I found the one I am looking for)

Our last terms need to be -3 and -7 since when multiplied together they equal 21 and when added together the sum is -10.

Step 4: Complete the binomials.

$(x - 3)\ (x - 7)$ ← Your Solution