Continuity Facts … Set 1

Continuity

Definition (Continuity). A function f is **continuous at** $x = c$ if

(i) $f(c)$ exists.

(ii) $\lim_{x \to c} f(x)$ exists.

(iii) $\lim_{x \to c} f(x) = f(c)$.

Otherwise, f is **discontinuous at** $x = c$. Furthermore, if f is continuous at every x in the open interval (a, b), then f is continuous on (a, b). If f is continuous on (a, b) and

$$\lim_{x \to a^+} f(x) = f(a) \qquad \lim_{x \to b^-} f(x) = f(b),$$

then f is continuous on $[a, b]$.

Note. Graphically, this definition means that f is continuous on an interval if and only if the graph of f can be drawn with a single, unbreaking stroke.

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Example. Examples of continuous functions:

(1) Polynomials are continuous everywhere.

$$f(x) = x + 1 \qquad g(x) = x^2 + 3 \qquad h(x) = x^{100} - x$$

(2) Rational functions are continuous on their domains.

$$f(x) = \frac{x+1}{x-2} \qquad (-\infty, 2) \cup (2, \infty)$$

$$g(x) = \frac{x}{x^2 + 3x + 2} \qquad (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$$

(3) If f and g are continuous at $x = c$, then kf (k a real number), $f \pm g$, fg, and $\frac{f}{g}$ ($g(c) \neq 0$) are continuous at $x = c$.

(4) Trigonometric, exponential, and logarithmic functions are all continuous everywhere on their domain.