Continuity Facts … Set 2

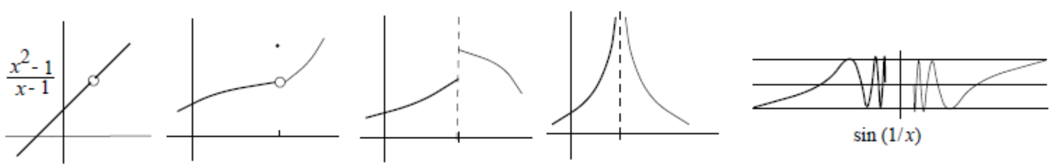
Continuity

To understand continuity, it helps to see how a function can fail to be continuous. All of the important functions used in calculus and analysis are continuous except at isolated points. Such points are called **points of discontinuity**. There are several types. Let's begin by first recalling the definition of continuity (cf. book, p. 75).

(2) $f(x)$ is **continuous** at a if $\lim_{x \to a} f(x) = f(a)$.

Thus, if a is a point of discontinuity, something about the limit statement in (2) must fail to be true.

Types of Discontinuity



removable *removable* *jump* *infinite* *essential*

In a **removable** discontinuity, $\lim_{x \to a} f(x)$ exists, but $\lim_{x \to a} f(x) \neq f(a)$. This may be because $f(a)$ is undefined, or because $f(a)$ has the "wrong" value. The discontinuity can be removed by changing the definition of $f(x)$ at a so that its new value there is $\lim_{x \to a} f(x)$. In the left-most picture, $\frac{x^2-1}{x-1}$ is undefined when $x = 1$, but if the definition of the function is completed by setting $f(1) = 2$, it becomes continuous — the hole in its graph is "filled in".

In a **jump** discontinuity (Example 2), the right- and left-hand limits both exist, but are not equal. Thus, $\lim_{x \to a} f(x)$ does not exist, according to (1). The *size* of the jump is the difference between the right- and left-hand limits (it is 2 in Example 2, for instance). Though jump discontinuities are not common in functions given by simple formulas, they occur frequently in engineering — for example, the square waves in electrical engineering, or the sudden discharge of a capacitor.

In an **infinite** discontinuity (Examples 3 and 4), the one-sided limits exist (perhaps as ∞ or $-\infty$), and at least one of them is $\pm\infty$.

An **essential** discontinuity is one which isn't of the three previous types — at least one of the one-sided limits doesn't exist (not even as $\pm\infty$). Though $\sin(1/x)$ is a standard simple example of a function with an essential discontinuity at 0, in applications they arise rarely, presumably because Mother Nature has no use for them.

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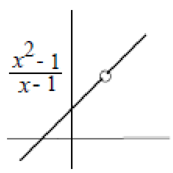
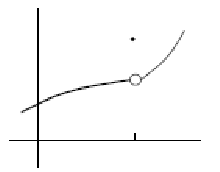


removable



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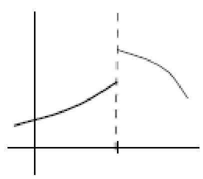

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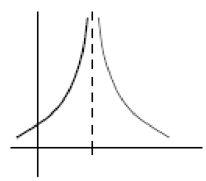

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In an **infinite** discontinuity the one-sided limits exist (perhaps as ∞ or $-\infty$), and at least one of them is $\pm\infty$.

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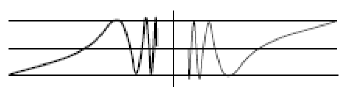

$\sin(1/x)$

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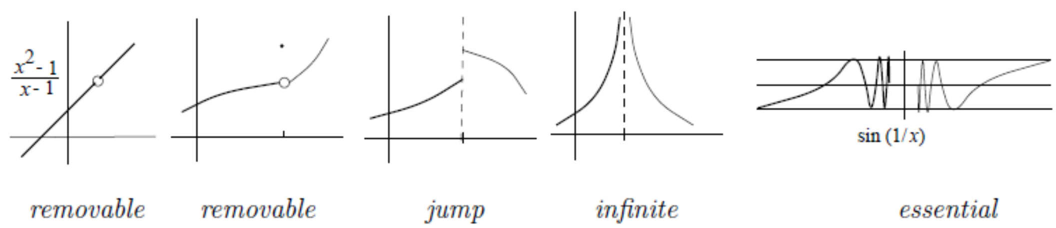

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