Continuity Facts … Set 4

Definition 1 We say the function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a).$$

Example We say that if $f(x)$ is a polynomial, then f is continuous at a for any real number a since $\lim_{x \to a} f(x) = f(a)$.

Note that this definition implies that the function f has the following three properties **if f is continuous at** a:

1. $f(a)$ is defined (a is in the domain of f).
2. $\lim_{x \to a} f(x)$ exists.
3. $\lim_{x \to a} f(x) = f(a)$.

Note that this implies that $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both exist and are equal.

Continuity Facts … Set 4

Types of Discontinuities

If a function f is defined near a (f is defined on an open interval containing a, except possibly at a), we say that f is discontinuous at a (or has a discontinuiuty at a) if f is not continuous at a. This can happen in a number of ways.

In the graph below, we have a catalogue of discontinuities. Note that a function is discontinuous at a if at least one of the properties 1-3 above breaks down.

Example 2 Consider the graph shown below of the function

$$k(x) = \begin{cases} x^2 & -3 < x < 3 \\ x & 3 \le x < 5 \\ 0 & x = 5 \\ x & 5 < x \le 7 \\ \frac{1}{x-10} & x > 7 \end{cases}$$

Where is the function discontinuous and why?