Continuity Practice … Set 3

For #1 – 5: Find the following Trigonometric Limits

3. $\lim_{x\to 0} \frac{\sin x}{x} =$

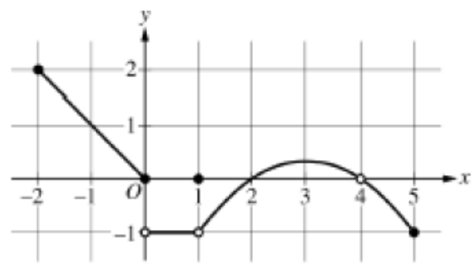
2. $\lim_{x\to 0} \frac{\cos x - 1}{x} =$

3. $\lim_{x\to 0} \frac{\sin 3x}{3x} =$

4. $\lim_{x\to 0} \frac{\sin 5x}{x} =$

5. $\lim_{x\to 0} \frac{\sin 7x}{5x} =$

For #6 – 7: Multiple choice. The graph of the function f is shown below.



Graph of f

6) For what value(s) of a does $\lim_{x\to a} f(x) =$ undefined?
   A) 0 and -2
   B) -2 and 5
   C) 1 and 5
   D) -2, 0, and 5

7) For what value(s) of a does $\lim_{x\to a} f(x) = -1$?
   A) 0 only
   B) 1 only
   C) 5 only
   D) 0, 1, and 5

Solutions:

1.) 1 2.) 0 3.) 1 4.) 5 5.) $^7/_5$ 6) D 7) B

2.6 wk

Do all work on your own paper!

For #1 – 8, discuss the continuity. If a discontinuity exists, then describe the type of discontinuity and its physical feature on a graph.

1) $f(x) = \frac{x^2-9}{x^2-4x+3}$

2) $g(x) = \frac{|x-3|}{x-3}$

3) $h(x) = \begin{cases} 3x-2; x > 3 \\ 5x^2 - e^{x-3}; x \le 3 \end{cases}$

4) $p(x) = \begin{cases} \sin 3x\,; x < 0 \\ x^2 - 4x; x > 0 \end{cases}$

5) $a(x) = \begin{cases} x-2; x \ne 1 \\ 6x-2; x = 1 \end{cases}$

6) $d(x) = \begin{cases} \frac{x^2+2x-8}{x+4}; x \ne -4 \\ -6; x = -4 \end{cases}$

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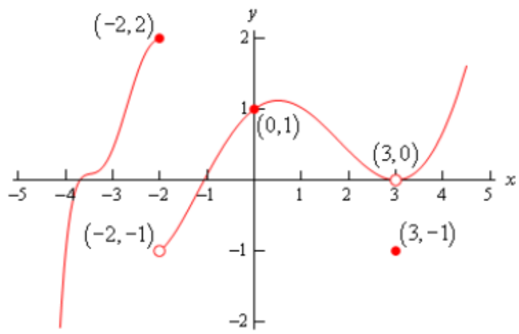
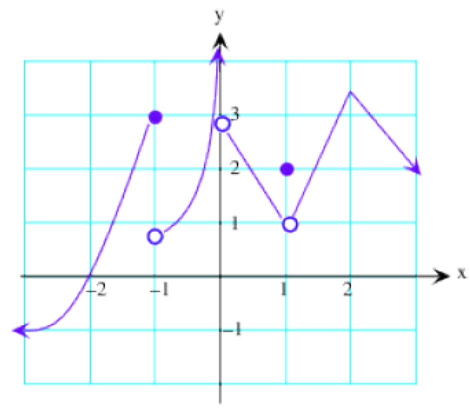
7) on the open interval (-4, 5)



8)



For #9 – 11: Use the definition of continuity to decide if $f(x)$ is continuous at the given value of x.

9) $f(x) = \begin{cases} \frac{x^2-4}{x+2}; & x < 2 \\ -4; & x = 2 \\ |x-4| - 2; & x > 2 \end{cases}$ 10) $h(x) = \begin{cases} x; x > 1 \\ x^2; x \le 1 \end{cases}$ 11) $h(x) = \begin{cases} -2x; x < 2 \\ x^2 - 4x; x > 2 \end{cases}$

For #12 – 14: Find the constant a, or the constants a and b, such that the function is continuous everywhere.

12) $h(x) = \begin{cases} x^3; x \le 2 \\ ax^2; x > 2 \end{cases}$ 13) $f(x) = \begin{cases} 2; & x \le -1 \\ ax + b; & -1 < x < 3 \\ -2; & x \ge 3 \end{cases}$

14) $g(x) = \begin{cases} \frac{x^2+3x+2}{x+1}; x \ne -1 \\ a; x = -1 \end{cases}$

For #15 – 16: Does the IVT guarantee a zero in the function over the indicated closed interval? Why or why not?

15) $f(x) = x^2 + x - 1, [0, 5]$ 16) $q(x) = x^2 - 6x + 2, [-1, -2]$

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Answers

1) Removable discontinuity (hole) at $x = 3$; non-removable discontinuity (VA) at $x = 1$

2) Non-removable discontinuity (jump) at $x = 3$

3) Non-removable discontinuity (jump) at $x = 3$

4) Removable discontinuity (hole) at $x = 0$

5) Removable discontinuity (hole) at $x = 1$

6) Continuous everywhere (no discontinuities)

7) Removable discontinuity (hole) at $x = 3$; non-removable discontinuity (jump) at $x = -2$

8) Removable discontinuity (hole) at $x = 1$; non-removable discontinuity (jump) at $x = -1$; non-removable discontinuity (VA) at $x = 0$… note that there is also a hole at $x = 0$ from the right side.

9) Not continuous; $\lim_{x \to 2-} f(x) = \lim_{x \to 2+} f(x) \neq f(2)$. (You must provide numerical evidence)

10) Continuous: $\lim_{x \to 1-} h(x) = \lim_{x \to 1+} h(x) = h(1)$. (You must provide numerical evidence)

11) Not continuous; $h(2)$ is not defined: $\lim_{x \to 2-} h(x) = \lim_{x \to 2+} h(x) \neq h(2)$.

12) 2 13) $a = -1; b = 1$ 14) 1

15) Yes; Because $f(x)$ is continuous on the closed interval [0, 5], $f(0) = -1$ and $f(5) = 29$, and 0 is between -1 and 29, by the IVT, $f(x)$ must equal zero at least once on this interval.

16) No; 0 is not between $f(-1)$and $f(-2)$, which have values of 9 and 18, respectively, and so the IVT does not apply.

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For #1 – 15, find each limit, if possible.

1) $\lim_{x\to\infty} \frac{-3x^2+5x^3+10}{x^2}$

2) $\lim_{x\to\infty} \frac{-3x^2+5x^3+10}{x^4}$

3) $\lim_{x\to\infty} \frac{-3x^2+5x^3+10}{x^3}$

4) $\lim_{x\to\infty} \frac{7x^2+2}{x^3-1}$

5) $\lim_{x\to\infty} \frac{x^2+2}{-2x^2-1}$

6) $\lim_{x\to-\infty} \frac{2x^3+5}{3x^3-1}$

7) $\lim_{x\to-\infty} \frac{2x^2+5}{3x^2-1}$

8) $\lim_{x\to\infty} \frac{6x-1}{10-8x}$

9) $\lim_{x\to-\infty} \frac{6x-1}{10-8x}$

10) $\lim_{x\to-\infty} \frac{x}{x^2}$

11) $\lim_{x\to\infty} \frac{5x^2-1}{3-2x}$

12) $\lim_{x\to\infty} \frac{5x}{\sqrt{4x^2-3x}}$

13) $\lim_{x\to-\infty} \frac{5x}{\sqrt{4x^2-3x}}$

14) $\lim_{x\to\infty} \frac{\sqrt{36x^2-7x}}{4-3x}$

15) $\lim_{x\to-\infty} \frac{\sqrt{36x^2-7x}}{4-3x}$

For #16 – 18, identify any asymptotes and the *x*-coordinates for any holes for each function.

16) $y = \frac{2+x}{1-x}$

17) $f(x) = \frac{2x+4}{x^2-4}$

18) $g(x) = \frac{x^2-3x-10}{x^2-25}$

19) $\lim_{x\to 0} \frac{e^x+\cos x-2x}{x^2-2}$

A) -1 B) 0 C) $\frac{1}{2}$
D) 1 E) nonexistent

20) $\lim_{x\to 0} \frac{\sin x \cos x}{x}$

A) -1 B) 0 C) 1
D) $\frac{\pi}{4}$ E) nonexistent

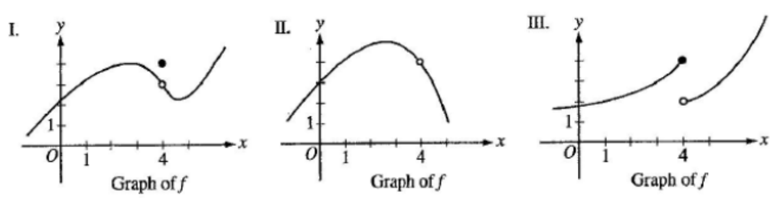
21) $\lim_{x\to\infty} \left(\frac{x^{17}-3x+2}{4\ln x}\right)$

22) $\lim_{x\to\infty} \left(\frac{-2\ln x}{x^4+5x^2}\right)$

23) $\lim_{x\to\infty} \left(\frac{-2e^x}{x^{55}}\right)$

24) For which of the following does $\lim_{x\to 4} f(x)$ exist?

A) I only
B) II only
C) III only
D) I and II only
E) I and III only



Continuity Practice … Set 3

Answers

1) DNE (∞) 2) 0 3) 5 4) 0 5) $-\frac{1}{2}$ 6) $\frac{2}{3}$ 7) $\frac{2}{3}$

8) $-\frac{3}{4}$ 9) $-\frac{3}{4}$ 10) 0 11) DNE ($-\infty$) 12) $\frac{5}{2}$ 13) $-\frac{5}{2}$ 14) -2

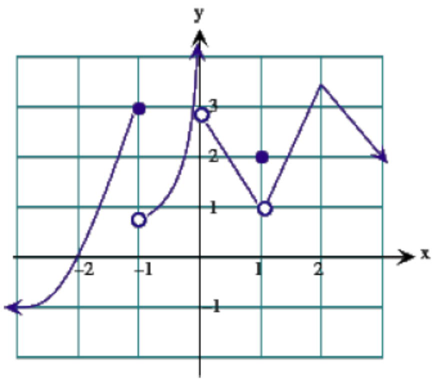
15) 2 16) VA at $x = 1$; HA at $y = -1$ 17) hole at $x = -2$; VA at $x = 2$; HA at $y = 0$

18) hole at $x = 5$; VA at $x = -5$; HA at $y = 1$ 19) A 20) C 21) DNE (∞) 22) 0

23) DNE ($-\infty$) 24) D

Continuity Practice … Set 3

Ch 2 Review Worksheet: Do all work on a separate piece of paper. No calculators are allowed unless a problem is marked with an asterisk. Show all work for credit.



1) Use the graph shown of $f(x)$ to find each limit, if possible.
   a) $\lim_{x\to 1} f(x)$ b) $\lim_{x\to -1} f(x)$
   c) $\lim_{x\to -1^-} f(x)$ d) $\lim_{x\to 0^-} f(x)$

For #2 – 6, find the limit, if possible.

2) $\lim_{x\to 4} \sqrt{x+2}$

3) $\lim_{t\to -2} \frac{t+2}{t^2-4}$

4) $\lim_{x\to 0} \frac{\sqrt{4+x}-\sqrt{4}}{x}$

5) $\lim_{x\to 0} \frac{1-\cos x}{x}$

6) $\lim_{x\to \pi/4} \frac{4x}{\tan x}$

7) Given that $\lim_{x\to c} f(x) = -\frac{3}{4}$ and that $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c}[f(x) + 2g(x)]$.

8) Given that $f(x) = 3x^2$ and $g(x) = \frac{1}{x-75}$, find all values of x where $g(f(x))$ is continuous.

9) Find the limit, if possible: $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

10) Find the limit, if possible: $\lim_{x\to 3^-} \frac{|x-3|}{x-3}$

11) Find $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} (x-2)^2 & \text{if } x \le 2 \\ 2-x & \text{if } x > 2 \end{cases}$

12) Find $\lim_{x\to 1^+} g(x)$, where $g(x) = \begin{cases} \sqrt{x-1} & \text{if } x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

13) Find $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3+1 & \text{if } x < 1 \\ \frac{1}{2}(x+3) & \text{if } x > 1 \end{cases}$

14) Determine the intervals on which the function is continuous: $f(x) = \frac{3x^2-x-2}{x-1}$

15) Determine the value of c such that the function is continuous everywhere:

$$h(x) = \begin{cases} x+3, & x \le 2 \\ cx+6, & x > 2 \end{cases}$$

16) Given that $f(3) = 7$, explain why you cannot conclude that $\lim_{x\to 3} f(x) = 7$.

17) Explain why $f(x) = 2x^3 - 3$ must have at least one zero on the interval [1, 2]. Do not use a calcula

18) Write the equations for any horizontal and vertical asymptotes: $a(x) = \frac{3x^2-6x}{x^2-4}$.

*19) Find the limit, if possible: $\lim_{x\to 1^-} \frac{x^2+2x+1}{x-1}$

*20) Find the limit, if possible: $\lim_{x\to -2^-} \frac{2x^2+x+1}{x+2}$

21) Find the limit, if possible: $\lim_{x\to 0} \frac{\sin 4x}{5x}$

22) Use the definition of continuity to decide whether or not $f(x)$ is continuous at $x = 5$.

$$f(x) = \begin{cases} x^2 + \ln(6-x) - 2, & x < 5 \\ 2x+13, & x \ge 5 \end{cases}$$

23) Use the definition of continuity to decide whether or not $g(x)$ is continuous at $x = -2$.

$$g(x) = \begin{cases} x^3+4, & x < -2 \\ 2x, & x > -2 \end{cases}$$

24) Find the limit, if possible: $\lim_{x\to \infty} \frac{2x^2}{3x^2+5}$

25) Find the limit, if possible: $\lim_{x\to -\infty} \frac{3\sqrt{4x^2-5}}{4x+5}$

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26) $\lim\limits_{x \to -\infty} \frac{\sqrt{x^2 + x}}{-2x}$

27) $\lim\limits_{x \to \infty} \frac{-17x}{3x^2 + 20}$

28) $\lim\limits_{x \to \infty} \frac{x^5}{9x-1}$

29) At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

A) undefined but not continuous

B) continuous (and thus defined)

C) defined by not continuous

D) undefined by continuous

30) If $\lim\limits_{x \to 3} f(x) = 7$ then which of the following must be true?

A) f is continuous at $x = 3$.

B) f is defined at $x = 3$.

C) Both A and B.

D) Neither A nor B.

Continuity Practice … Set 3

Answers

1a) 1   1b) DNE   1c) 3   1d) ∞ (DNE) 2) $\sqrt{6}$

3) $-\frac{1}{4}$ 4) $\frac{1}{4}$ 5) 0   6) π 7) $\frac{7}{12}$

8) $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$ 9) $\frac{1}{4}$ 10) -1   11) 0

12) 2   13) 2   14) $(-\infty,1)\cup(1,\infty)$ 15) $-\frac{1}{2}$

16) We do not know what the function is *approaching* from the left and right sides, so we cannot make a conclusion about a limit.

17) Since the function is continuous on a closed interval, $f(1)$ is negative, and $f(2)$ is positive, by the IVT, the function must cross the x-axis, and thus have a zero on this interval.

18) HA at $y = 3$; VA at $x = -2$ 19) $-\infty$ (DNE) 20) $-\infty$ (DNE) 21) $\frac{4}{5}$

22) Yes, $f(x)$ is continuous at $x = 5$ because $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$.

23) No, $f(x)$ is not continuous, because it is not defined at $x = 2$.

24) $\frac{2}{3}$ 25) $-\frac{3}{2}$ 26) $\frac{1}{2}$ 27) 0   28) ∞ (DNE)

29) B   30) D