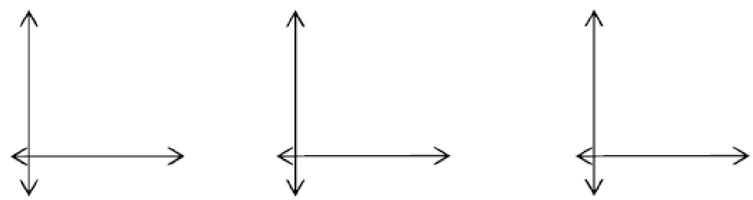
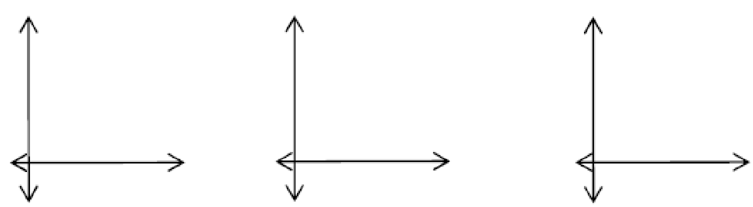
Limits Rules … Set 2

Definitions of Limits at Large Numbers

	Definition in Words	Precise Mathematical Definition
Large POSITIVE numbers	Let *f* be a function defined on some interval (a, ∞). Then $\lim_{x\to\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large in a positive direction.	Let f be a function defined on some interval (a, ∞). Then $\lim_{x\to\infty} f(x) = L$ if for every $\varepsilon > 0$ there is a corresponding number N such that if $x > N$ then $\lvert f(x) - L \rvert < \varepsilon$
Large NEGATIVE numbers	Let *f* be a function defined on some interval $(-\infty, a)$. ∞). Then $\lim_{x\to-\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large in a negative direction.	Let f be a function defined on some interval $(-\infty, a)$. Then $\lim_{x\to-\infty} f(x) = L$ if for every $\varepsilon > 0$ there is a corresponding number N such that if $x < N$ then $\lvert f(x) - L \rvert < \varepsilon$

	Definition	What this can look like…
Horizontal Asymptote	The line $y = L$ is a **horizontal asymptote** of the curve $y = f(x)$ if either is true: 1. $\lim_{x\to\infty} f(x) = L$ *or* 2. $\lim_{x\to-\infty} f(x) = L$	
Vertical Asymptote	The line $x = a$ is a **vertical asymptote** of the curve $y = f(x)$ if *at least one* of the following is true: 1. $\lim_{x\to a} f(x) = \infty$ 2. $\lim_{x\to a^-} f(x) = \infty$ 3. $\lim_{x\to a^+} f(x) = \infty$ 4. $\lim_{x\to a} f(x) = -\infty$ 5. $\lim_{x\to a^-} f(x) = -\infty$ 6. $\lim_{x\to a^+} f(x) = -\infty$	

Theorem

- If $r > 0$ is a rational number then $\lim_{x\to\infty} \frac{1}{x^r} = 0$
- If $r > 0$ is a rational number such that x^r is defined for all x then $\lim_{x\to-\infty} \frac{1}{x^r} = 0$