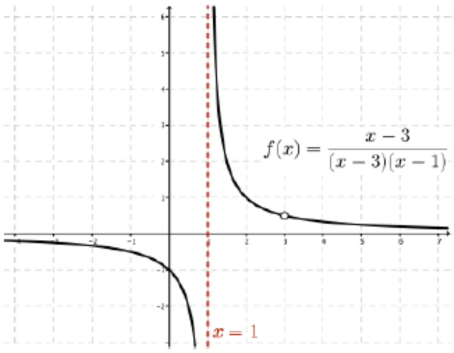
Exploring Limits … Set 2

For the function f whose graph is given, state the value of the given quantity, if it exists. Identify any discontinuities in the graph of f.

1.



Domain: Range:

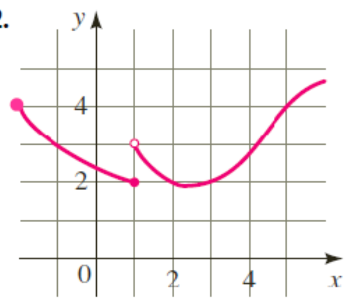
$\lim_{x \to 1^-} f(x)$ = _____ $\lim_{x \to 3} f(x)$ = _____

$\lim_{x \to 1^+} f(x)$ = _____ $f(3)$ = _____

$\lim_{x \to 1} f(x)$ = _____ $f(1)$ = _____

$\lim_{x \to \infty} f(x)$ = _____ $\lim_{x \to -\infty} f(x)$ = _____

2.



Domain: Range:

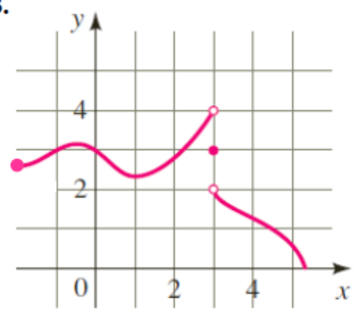
$\lim_{x \to 1^-} f(x)$ = _____ $\lim_{x \to -1} f(x)$ = ______

$\lim_{x \to 3} f(x)$ = ______ $\lim_{x \to -2} f(x)$ = ______

$\lim_{x \to 1} f(x)$ = ______ $\lim_{x \to 5} f(x)$ = ______

$f(1)$ = ______ $f(5)$ = ______

3.



Domain: Range:

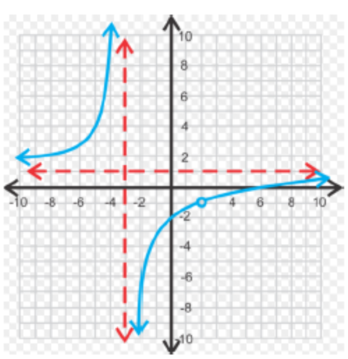
$\lim_{x \to 3^-} f(x)$ = _____ $\lim_{x \to 0} f(x)$ = ______

$\lim_{x \to 3^+} f(x)$ = ______ $\lim_{x \to -1} f(x)$ = ______

$\lim_{x \to 3} f(x)$ = ______ $f(-1)$ = _______

$f(3)$ = ______ $f(0)$ = ______

4.



Domain: Range:

$\lim_{x \to -3^+} f(x)$ = ______ $\lim_{x \to -3^-} f(x)$ = ______

$\lim_{x \to -\infty} f(x)$ = ______ $\lim_{x \to \infty} f(x)$ = ______

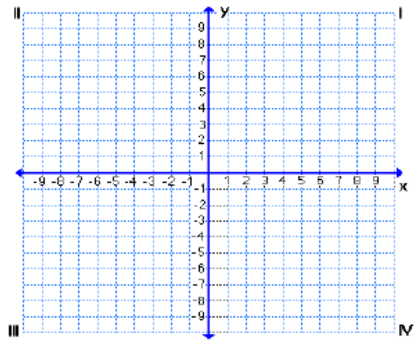
$\lim_{x \to 2} f(x)$ = ______ $\lim_{x \to -3} f(x)$ = ______

Exploring Limits … Set 2

Graph the piecewise-defined function, then state the value of the given quantity, if it exists. Identify any discontinuities in the graph of f.

5. $f(x) = \begin{cases} x^2 - 1 & if\ x \le 2 \\ -2x + 7 & if\ x > 2 \end{cases}$

Discontinuities:



$\lim_{x \to 2^-} f(x) =$ ________

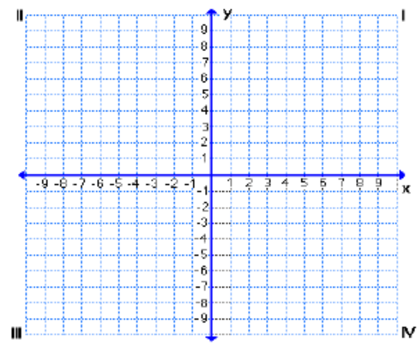
$\lim_{x \to 2^+} f(x) =$ ________

$\lim_{x \to 2} f(x) =$ ________

$f(2) =$ _________

6. $f(x) = \begin{cases} x + 3 & if\ x < -2 \\ -x^2 & if\ x \ge -2 \end{cases}$

Discontinuities:



$\lim_{x \to -2^-} f(x) =$ ________

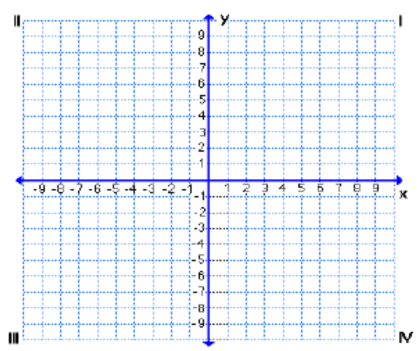
$\lim_{x \to -2^+} f(x) =$ ________

$\lim_{x \to -2} f(x) =$ ________

$f(-2) =$ _________

7. $f(x) = \begin{cases} 2x + 2 & if\ x \ne 1 \\ 7 & if\ x = 1 \end{cases}$

Discontinuities:



$\lim_{x \to 1} f(x) =$ _________

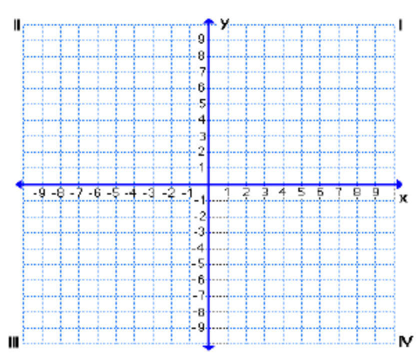
$\lim_{x \to 3} f(x) =$ _________

$\lim_{x \to 0} f(x) =$ _________

$f(1) =$ _________

8. $f(x) = \begin{cases} 2x + 2 & if\ x < 1 \\ 7 & if\ x > 1 \end{cases}$

Discontinuities:



$\lim_{x \to 1^-} f(x) =$ _________

$\lim_{x \to 1^+} f(x) =$ _________

$\lim_{x \to 1} f(x) =$ _________

$f(1) =$ _________

Exploring Limits … Set 2

Graph the rational function. Find all intercepts and asymptotes. Then state the value of the limit, if it exists.

9. $f(x) = \frac{x^3+3x^2-16x-48}{x^2+2x-3}$

y-intercept:

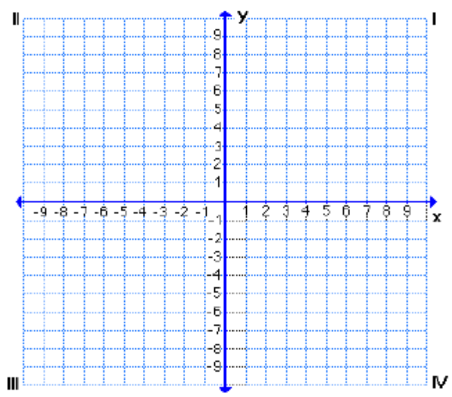
x-intercept(s):

Horizontal Asymptote:

Slant Asymptote:

Vertical Asymptote(s):

Hole(s) in the graph:



$\lim_{x\to -3} f(x) =$ ________

$\lim_{x\to 1} f(x) =$ ________

$\lim_{x\to 1^+} f(x) =$ ________

$\lim_{x\to 0} f(x) =$ ________

$\lim_{x\to \infty} f(x) =$ ________

$\lim_{x\to -\infty} f(x) =$ ________

10. $f(x) = \frac{x^2-3x-10}{x^2-2x-15}$

y-intercept:

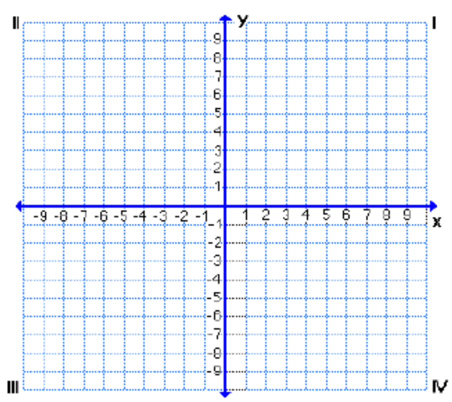
x-intercept(s):

Horizontal Asymptote:

Slant Asymptote:

Vertical Asymptote(s):

Hole(s) in the graph:



$\lim_{x\to -3^-} f(x) =$ ________

$\lim_{x\to 5} f(x) =$ ________

$\lim_{x\to -3} f(x) =$ ________

$\lim_{x\to 15} f(x) =$ ________

$\lim_{x\to \infty} f(x) =$ ________

$\lim_{x\to -\infty} f(x) =$ ________

Exploring Limits … Set 2

Evaluate the limit if it exists, or state "does not exist."

11. $\lim_{h \to 0} \frac{(3x+h)^3 - 27x^3}{h}$

12. $\lim_{x \to -\infty} (5x^2 - 2x + 3)$

13. $\lim_{x \to -2} \frac{x^4 + 2x^3 - 2x^2 - 3x + 2}{x^3 + 5x^2 - 4x - 20}$

14. $\lim_{x \to 5} \frac{3x^2 - 13x - 10}{2x - 10}$

15. $\lim_{x \to 0} \frac{(4+x)^2 - 16}{x}$

16. $\lim_{x \to -\infty} \frac{5x^3 - 2}{4x^3 + x + 5}$

17. $\lim_{x \to \infty} \frac{x+8}{x-5}$

18. $\lim_{x \to 5} \frac{x+8}{x-5}$

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19. $\lim_{x \to -2} \frac{x^3-3x+2}{x^3+8}$

20. $\lim_{x \to 5^+} \frac{x+8}{x-5}$

21. $\lim_{x \to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$

22. $\lim_{x \to -3^-} \frac{x-1}{x+3}$

23. $\lim_{x \to -3} \frac{\sqrt{x+7}-2}{x+3}$

24. $\lim_{x \to \infty} \frac{x+5}{x^2-4}$

25. $\lim_{x \to 2} \frac{x+5}{x^2-4}$

26. $\lim_{x \to 1} \frac{2x^3+3x^2-8x+3}{x^2-3x+2}$

Exploring Limits … Set 2

27. $\lim\limits_{x \to \infty} \frac{4}{5^{\frac{1}{x}}}$

28. $\lim\limits_{x \to -5} \frac{2x^3 - 42x + 40}{x^3 + 5x^2}$

29. $\lim\limits_{x \to 4} \frac{x^4 - 15x^2 - 16}{x^3 - 64}$

30. $\lim\limits_{x \to 2} \frac{x^4 - x^2 - 12}{x^3 - 2x^2 - x + 2}$

31. $\lim\limits_{x \to -2} \sqrt{x^4 + 3x + 6}$

32. $\lim\limits_{x \to 4} \frac{2x^2 - 5x - 12}{3x^2 - 11x - 4}$

33. $\lim\limits_{h \to 0} \frac{(h+3)^4 - 81}{h}$

34. $\lim\limits_{h \to 0} \frac{(2+h)^5 - 32}{h}$

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35. $\lim_{x \to 4} (5x^2 - 2x + 3)$

36. $\lim_{h \to 0} \frac{[(x+h)^2 - 3(x+h)] - (x^2 - 3x)}{h}$

37. $\lim_{h \to 0} \frac{[2(x+h)^2 + (x+h)] - (2x^2 + x)}{h}$

38. $\lim_{x \to 0} \frac{\frac{1}{x+7} - \frac{1}{7}}{x}$

39. $\lim_{x \to 0} \frac{(x-5)^{-1} + 5^{-1}}{x}$

40. $\lim_{x \to 16} \frac{4 - \sqrt{x}}{x - 16}$

41. $\lim_{x \to \frac{2}{3}} (27x^3 - 8)(3x^2 - 2x)^{-1}$

42. $\lim_{x \to 3} \frac{x^2}{x - 3}$

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43. $\lim_{x \to 3^+} \frac{x}{x-3}$

44. $\lim_{x \to \infty} \frac{x^2}{x-3}$

45. $\lim_{x \to 1} \frac{x+2}{x^2-1}$

46. $\lim_{x \to \infty} \frac{x+2}{x^2-1}$

47. $\lim_{x \to 0} \frac{(x+3)^{-1} - 3^{-1}}{x}$

48. $\lim_{x \to 0} [(x+2)^2 - 4](x+4)^{-1}$

49. Given $f(x) = \frac{x^3 + x^2 - 4x - 4}{x^2 + x - 2}$

a. Describe the differences between the following limits.

$\lim_{x \to -2} f(x) =$ $\lim_{x \to 1} f(x) =$ $\lim_{x \to 0} f(x) =$ $\lim_{x \to -\infty} f(x) =$ $\lim_{x \to \infty} f(x) =$

b. Why do some exist and some not exist?

c. Under what circumstances would $\lim_{x \to 1} f(x)$ exist?

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50. Given $f(x) = \sqrt{x}$, why does the limit as x approaches zero not exist?

51. Given a function, $f(x)$, if $\lim_{x\to 2} f(x) = 4$ and $f(2) = -3$, which of the following is true?

a. There is a point discontinuity at $x = 2$
b. There is a jump discontinuity at $x = 2$
c. There is a line discontinuity at $x = 2$
d. The function is continuous at $x = 2$

52. Given a function, $f(x)$, if $\lim_{x\to -3} f(x) = 5$ and $f(-3) = 5$, which of the following is true?

a. There is a point discontinuity at $x = -3$
b. There is a jump discontinuity at $x = -3$
c. There is a line discontinuity at $x = -3$
d. The function is continuous at $x = -3$

53. Given a function, $f(x)$, if $\lim_{x\to 4^-} f(x) = 3$, $\lim_{x\to 4^+} f(x) = -1$ and $f(4) = 9$, which of the following is true?

a. There is a point discontinuity at $x = 4$
b. There is a jump discontinuity at $x = 4$
c. There is a line discontinuity at $x = 4$
d. The function is continuous at $x = 4$

54. Given a function, $f(x)$, if $\lim_{x\to 0^-} f(x) = -\infty$, $\lim_{x\to 0^+} f(x) = -\infty$ and $f(0)$ is undefined, which of the following is true?

a. There is a point discontinuity at $x = 0$
b. There is a jump discontinuity at $x = 0$
c. There is a line discontinuity at $x = 0$
d. The function is continuous at $x = 0$