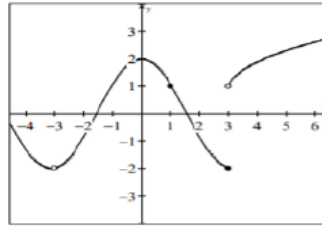


# One-sided Limits ... Set 1

## One-Sided Limits Graphically and Algebraically

1. Given the graph of  $f(x)$ , determine the following.



a.  $\lim_{x \rightarrow -3^-} f(x)$

b.  $\lim_{x \rightarrow -3^+} f(x)$

c.  $\lim_{x \rightarrow -3} f(x)$

d.  $\lim_{x \rightarrow 1^-} f(x)$

e.  $\lim_{x \rightarrow 1^+} f(x)$

f.  $\lim_{x \rightarrow 1} f(x)$

g.  $\lim_{x \rightarrow 3^-} f(x)$

h.  $\lim_{x \rightarrow 3^+} f(x)$

i.  $\lim_{x \rightarrow 3} f(x)$

j.  $f(-3)$

k.  $f(1)$

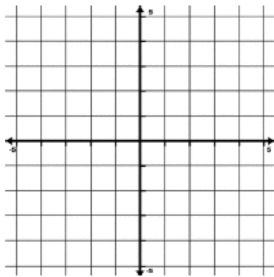
l.  $f(3)$

2. Sketch each piecewise function below and determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

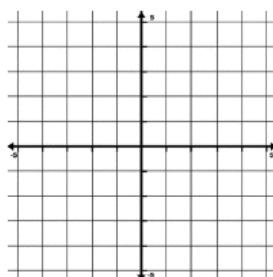
a.  $f(x) = \begin{cases} 2, & x < 1 \\ 3, & x = 1 \\ x + 1, & x > 1 \end{cases}$

b.  $f(x) = \begin{cases} 4 - x^2, & -2 < x \leq 2 \\ x - 2, & x > 2 \end{cases}$

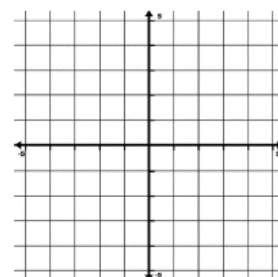
c.  $f(x) = \begin{cases} |x + 2| + 1, & x < -1 \\ -x + 1, & -1 \leq x \leq 1 \\ x^2 - 2x + 2, & x > 1 \end{cases}$



$\lim_{x \rightarrow 1} f(x)$



$\lim_{x \rightarrow 2} f(x)$



$\lim_{x \rightarrow 1} f(x)$

3. For each function below, determine, if it exists, the given limit. If the limit does not exist, provide an explanation.

a.  $f(x) = \begin{cases} 2x - 1, & x \leq -2 \\ -x + 2, & x > -2 \end{cases}$

b.  $f(x) = \begin{cases} -x^2 + 4x - 3, & x < 1 \\ x - 7, & x \geq 1 \end{cases}$

Find  $\lim_{x \rightarrow -2^+} f(x)$

Find  $\lim_{x \rightarrow 1^-} f(x)$

# One-sided Limits ... Set 1

c. 
$$f(x) = \begin{cases} x + 3, & x \in (-\infty, 0] \\ -x + 2, & x \in (0, 2) \\ (x - 2)^2, & x \in [2, \infty) \end{cases}$$

Find  $\lim_{x \rightarrow 0} f(x)$  and  $\lim_{x \rightarrow 2} f(x)$

d. 
$$f(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ -\frac{x}{2} + \frac{7}{2}, & x \geq -1 \end{cases}$$

Find  $\lim_{x \rightarrow -1} f(x)$

e. 
$$f(x) = \begin{cases} (x + 1)^2 - 1, & -2 \leq x < 0 \\ \frac{5}{4} \sin\left(\frac{\pi x}{2}\right), & 0 \leq x < 2 \\ (x - 3)^2 - 1, & 2 \leq x \leq 4 \end{cases}$$

Find  $\lim_{x \rightarrow 2} f(x)$

Evaluate each limit.

4.  $\lim_{x \rightarrow 2^+} \frac{x}{x-2}$

5.  $\lim_{x \rightarrow -3^+} \frac{x+1}{x^2-6x+9}$

6.  $\lim_{x \rightarrow -3^-} \frac{x+2}{x^2+6x+9}$

7.  $\lim_{x \rightarrow 2^+} \frac{x-2}{x^2+4x+4}$

8.  $\lim_{x \rightarrow -3^-} \frac{x^2}{3x+9}$

9.  $\lim_{x \rightarrow 2^+} \frac{x^2}{2x-4}$

10.  $\lim_{x \rightarrow -2^+} \frac{1}{x^2-4}$

11.  $\lim_{x \rightarrow 1^-} -\frac{2}{x^2-1}$