# Formal Definition of Derivative … Facts 1

**Limit-Definition of the Derivative**: $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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**Example:**

Find the derivative of $f(x) = x^2 - 4x$ .

So $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{[(x+h)^2 - 4(x+h)] - [x^2 - 4x]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}$$

$$= \lim_{h \to 0} (2x + h - 4)$$

$$= 2x - 4 .$$

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**Example:** Find the derivative of $f(x) = x^3 + 5x^2 - 4$.

$$\text{Now } f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^3 + 5(x+h)^2 - 4] - [x^3 + 5x^2 - 4]}{h}$$

$$= \lim_{h \to 0} \frac{(x^3 + 3xh^2 + 3x^2h + h^3) + (5x^2 + 10xh + 5h^2) - 4 - x^3 - 5x^2 + 4}{h}$$

$$= \lim_{h \to 0} \frac{3xh^2 + 3x^2h + h^3 + 10xh}{h}$$

$$= \lim_{h \to 0} \frac{h(3xh + 3x^2 + h^2 + 10x)}{h}$$

$$= \lim_{h \to 0} (3xh + 3x^2 + h^2 + 10x)$$

$$= 3x^2 + 10x$$

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2. Find the derivative of each function using the limit definition.

(a) $f(x) = x^2 + 3x - 5$

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2. Find the derivative of each function using the limit definition.

(b) $f(x) = 2x^2 + 7x$

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2. Find the derivative of each function using the limit definition.

(c) $f(x) = 4x^3 - 6x$

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3. Find the derivative of each function using the limit definition.

(a) $f(x) = \sqrt{x}$

(b) $f(x) = \dfrac{1}{x}$

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## Answers

1. $4x + 2h + 7$

2. (a) $f'(x) = 2x + 3$ (b) $f'(x) = 4x + 7$ (c) $f'(x) = 12x^2 - 6$

3. (a) $f'(x) = \dfrac{1}{2\sqrt{x}}$ (b) $f'(x) = -\dfrac{1}{x^2}$